Investigation of Stability Issues introduced by Network Reduction in EMT Simulations

Kasun Samarawickrama, Anuradha Kariyawasam, Sachintha Kariyawasam

***Abstract*-- Electromagnetic Transient (EMT) simulation studies focus on transient behavior in a selected, small area (the study area) of a power system. In order to reduce the time and the effort required for modelling and computation, the rest of the system (the external area) is represented by a network equivalent. The mathematical process used to obtain the network equivalent can arbitrarily introduce physically unrealizable negative resistances in the reduced network. This is a commonly seen occurrence in network reduction of large power systems, which can cause unstable EMT simulations. This paper proposes a novel technique to eliminate negative resistances, which could be introduced during the network reduction process. The resulting network equivalent is guaranteed to have positive resistors; regardless of the number of negative resistors appeared in the original reduced network. One of the major advantages of this technique over the existing methods is that it eliminates all the negative resistances in a single iteration by increasing the order of the network equivalent. Results presented in this paper indicate that the proposed method is efficient, robust and flexible to cater for additional network constraints, if required.**

***Keywords*: Negative resistances, network equivalent, network reduction, simulation stability**

I. Introduction

Size and complexity of contemporary power systems continue to grow as more and more power networks are interconnected and meshed up to meet the ever increasing economic and technical requirements. Electromagnetic Transient (EMT) simulations are a key tool used for investigation of power systems in both industry and academia. EMT simulations are performed essentially to obtain finer details of a power system such as high frequency transients in voltages and currents, which require detailed modelling of components and small simulation time-steps (typically 0.1-100 µs [1]). These higher frequency components produced by fast transient phenomena attenuate rapidly as they propagate in space. As a result, their effects remain localized and can only be observed in the vicinity of the disturbance. Therefore, in many EMT studies, precise and detailed representation is only required for a selected, smaller section of the larger power system, which is known as the study area. The rest of the network, referred to as the external area, is represented with simplified models and therefore, can be reduced into an equivalent of convenient proportions.

This process of network reduction [2], [3] is an integral step and a common practice in EMT modelling of large networks. This technique is regularly used in the industry for a number of different types of EMT studies such as transient recovery voltage (TRV) studies, temporary over voltage (TOV) studies and switching over voltage (SOV) studies.

Typically, a study area of an appropriate size is identified first and modelled in detail. For example, in a transient recovery voltage (TRV) study, the study area is selected up to 1 to 2 buses away from the circuit breaker under study [4]. In the next stage of the study, the external area is reduced into equivalent impedances and current injections at boundary buses. This is typically achieved using a power flow model where network is represented by lumped parameter. A simple Kron reduction algorithm [2], [3] is used for network reduction by removing nodes in the external area. Although simple (frequency independent) network equivalents are commonly used in typical EMT studies, Frequency Dependent Network Equivalents (FDNE) are required in special applications [1]. It is important to notice that the simplified modelling and the reduction of size of the external area has little effect on the events under investigation, once a study area of a suitable size has been chosen. Defining the study area is, therefore, a critical step in an EMT study that requires a certain degree of engineering judgment. Although the type of study has some bearing on this selection, it is largely at the discretion of the simulation engineer conduction the study. Furthermore, since time consumed by an EMT simulation is heavily dependent on the size and complexity of the network model, it is also desirable to model the required system in the simplest possible manner.

One of the inherent drawbacks of the above-mentioned procedure of network reduction is that it frequently introduces negative resistances to the network equivalent [5]. These negative resistances have no physical interpretation, as their existence is purely mathematical. Nonetheless, these artificial negative resistances can cause EMT simulations to become unstable [3]. This phenomenon is illustrated using a simple example in section IV.

There are several methods mentioned in literature to eliminate these negative resistances produced by the network reduction process. One of the methods is to ignore the negative resistance and adjust the power generation and loads to meet the expected power flow [6]. The trial-and-error nature of above method increases the computational time and sometimes adjustments on power generation may not be realistic. Another method is removing the negative resistors and injecting currents at the nodes where the negative resistors were connected [6]. In both of the above methods, power flow at the boundary buses can be matched, however, the short circuit levels are altered due

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to the elimination of the negative resistances. Another popular method is delta-star conversion [6]. A major drawback of this method is the addition of a node for every negative resistor in the reduced network. Further, this does not guarantee negative resistance elimination in its first attempt. In that case, an iterative approach is required until a given negative resistor is completely removed. In general, each occurrence of negative resistance has to be treated individually in all existing methods.

This paper presents a novel technique that introduces a single additional node to the original network equivalent and re-synthesizes it as a new, positive-real circuit, thus guaranteeing its stability. This method has a distinct advantage over the existing methods as it eliminates all the negative resistances from a single-step by increasing the order of the network equivalent only by one. The solution is obtained by solving a multi-variable, non-linear inequality problem yielding multiple solutions, which satisfy the stability requirements of EMT simulations. Therefore, further constraints can be added to the problem in such a manner that the solution space satisfies additional desirable requirements. As an example, a solution with a more realistic X/R ratio can be selected.

The rest of the paper is organized as follows. Section II explains the construction of network equivalent and how the existence of negative resistances in it affects the stability of EMT simulations. In section III, the proposed method is explained in detail with the associated mathematical formulation. Sections IV presents the results of the proposed method using a simple example. Section V follows with the conclusions of this study.

II. Network Equivalent and Stability

This section explains briefly the method of obtaining the network equivalent and the stability of such networks.

A. Network Reduction

Kron reduction is a commonly employed mathematical tool used in many power system studies to obtain the equivalent of a system with a large number of nodes [2], [7]. In its simplest form, Kron reduction eliminates all zero current injection nodes from the network to acquire the equivalent. However, this is often insufficient and it is required to reduce the network into an equivalent at nominated boundary buses. Following is the procedure of reducing a network of m nodes into a network of n nominated nodes (where, $m > n$), using Kron reduction.

A power system can be represented in admittance matrix form as,

$$\begin{bmatrix} I_1 \\ \vdots \\ I_n \\ \vdots \\ I_m \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} Y_{1,1} & \cdots & Y_{1,n} \\ \vdots & \ddots & \vdots \\ Y_{n,1} & \cdots & Y_{n,n} \end{bmatrix} & \begin{bmatrix} Y_{1,n+1} & \cdots & Y_{1,m} \\ \vdots & \ddots & \vdots \\ Y_{n,n+1} & \cdots & Y_{n,m} \end{bmatrix} \\ \begin{bmatrix} Y_{n+1,1} & \cdots & Y_{n+1,n} \\ \vdots & \ddots & \vdots \\ Y_{m,1} & \cdots & Y_{m,n} \end{bmatrix} & \begin{bmatrix} \cdots & \cdots & Y_{n+1,m} \\ \vdots & \ddots & \vdots \\ \cdots & \cdots & Y_{m,m} \end{bmatrix} \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_n \\ \vdots \\ V_m \end{bmatrix} \quad (1)$$

Re-writing (1) with sub-matrixes gives,

$$\begin{bmatrix} I_a \\ I_b \end{bmatrix} = \begin{bmatrix} A_{n\times n} & B_{(m-n)\times(m-n)} \\ C_{(m-n)\times(m-n)} & D_{n\times n} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} \quad (2)$$

This yields,

$$[I_a] = [A][V_a] + [B][V_b] \quad (3)$$
$$[I_b] = [C][V_a] + [D][V_b] \quad (4)$$

Substituting V_b from (4) in (3) yields,

$$[I_a] = [A][V_a] + [B][D]^{-1}[I_b] - [B][D]^{-1}[C][V_a] \quad (5)$$

Re-writing (5) gives,

$$[I_a'] = [Y_{bus,eq}][V_a] \quad (6)$$

Where,

$$[I_a'] = [I_a] - [B][D]^{-1}[I_b] \quad (7)$$
$$[Y_{eq}] = [A] - [B].[D]^{-1}[C] \quad (8)$$

Notice that the original m×m admittance matrix is now reduced to an equivalent matrix of n×n.

In the process of creating the original admittance matrix, all the network components were represented with their admittances at power frequency. Therefore, the reduced n×n equivalent matrix is also accurate at power frequency. Since the admittance matrix of the reduced network may contain negative resistances, it is worthwhile to further examine the stability of the network.

B. Stability of impedance networks

It has been shown in [8], the admittance matrix of a multi-port power network containing passive elements, which is linear time-invariant (LTI), is "positive-real". All elements of a positive-real matrix, Y(s), are stable and do not have poles in the right half s-plane [8], [9].

As explained in the earlier section, the process of network reduction only affects the external area of the modelled power system. The external area initially represented with lumped parameter models is understandably a passive and LTI system and hence, its admittance matrix is positive-real. Conversely, the admittance matrix of a reduced network containing negative resistances may cause instability, as it is no longer a positive-real matrix. Such instabilities can be eliminated by ensuring the resistances of the reduced network are non-negative.

III. Proposed Solution

The following procedure is proposed to eliminate negative resistances in a network equivalent. The network is first subjected to typical "Kron" reduction to obtain the reduced network. If the reduced network contains negative resistances, then an additional node (with zero current injection) will be introduced, such that the resulting network is equivalent to the reduced network and devoid of negative resistances (Fig. 1). The new network is required to satisfy the following conditions.

1. The equivalent impedance of the new network should be equal to that of the original network
2. The additional artificial node should have a zero current injection
3. Should not consist of any negative resistors

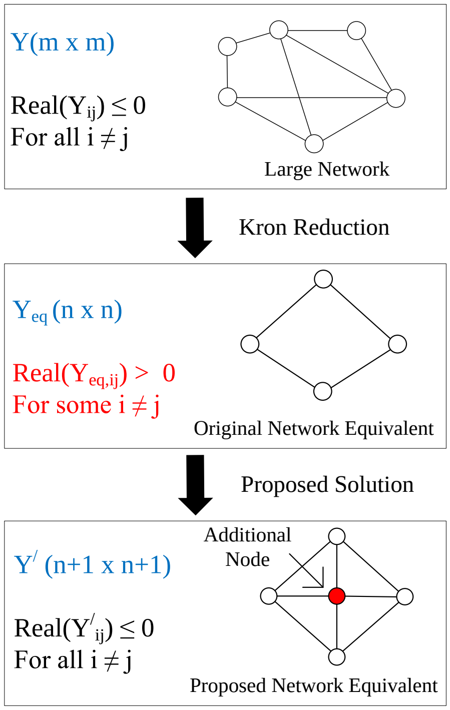

Fig. 1. An illustration of the derivation of proposed solution

Fig. 1 illustrates the proposed procedure of obtaining a network equivalent free of negative resistances. The admittance matrix of the original power system is given by Y, whereas that of the reduced network (using Kron reduction) is Y_{eq}. The admittance matrix of the proposed equivalent network with the additional node is Y'. The formulation of the inequalities which define the conditions for guaranteed positive resistances starts from Y'.

Y' can be written as follows,

$$Y' = \begin{bmatrix} Y'_{1,1} & \cdots & Y'_{1,n} & Y'_{1,n+1} \\ \vdots & \ddots & \vdots & \vdots \\ Y'_{n,1} & \cdots & Y'_{n,n} & Y'_{n,n+1} \\ Y'_{n+1,1} & \cdots & Y'_{n+1,n} & Y'_{n+1,n+1} \end{bmatrix} \tag{13}$$

Y'can be re-written with four sub-matrices as,

$$Y' = \begin{bmatrix} P & Q \\ R & S \end{bmatrix} \tag{14}$$

Where,

$$P = \begin{bmatrix} Y'_{1,1} & \cdots & Y'_{1,n} \\ \vdots & \ddots & \vdots \\ Y'_{n,1} & \cdots & Y'_{n,n} \end{bmatrix} \tag{15}$$

$$Q = \begin{bmatrix} Y'_{1,n+1} \\ \vdots \\ Y'_{n,n+1} \end{bmatrix} \tag{16}$$

$$R = [Y'_{n+1,1} \quad \cdots \quad Y'_{n+1,n+1}] \tag{17}$$

$$S = [Y'_{n+1,n+1}] \tag{18}$$

As per equations (7) and (8), Y_{eq} can be written in terms of P, Q, R and S sub-matrices as,

$$Y_{eq} = P - \frac{R.Q}{S} \quad or \quad P = Y_{eq} + \frac{R.Q}{S} \tag{19}$$

Therefore, to guaranteed positive resistances, P, Q, R and S sub-matrices should be calculated such that,

$$Real(Y'_{i,j}) \leq 0 \qquad for\ all\ i \neq j \tag{20}$$

$$Real(Y'_{i,i}) \geq Real\left(\sum_{j=1}^{n+1} -Y'_{i,j}\right) for\ all\ i \neq j \tag{21}$$

Above inequalities (20) and (21) define the required solution. This is a multivariable non-linear problem. Therefore, it gives multiple solutions (a solution space), which satisfy above inequalities. Hence, a more realistic solution can be selected at user's discretion. As an example, in power system point of view, a solution with a realistic X/R ratio is desirable.

IV. Example Case

The IEEE 39 bus system [10] is used to demonstrate the applicability of the proposed technique in the presence of negative resistances in network equivalents. The IEEE 39 bus system and the selected study area is shown in Fig. 2.

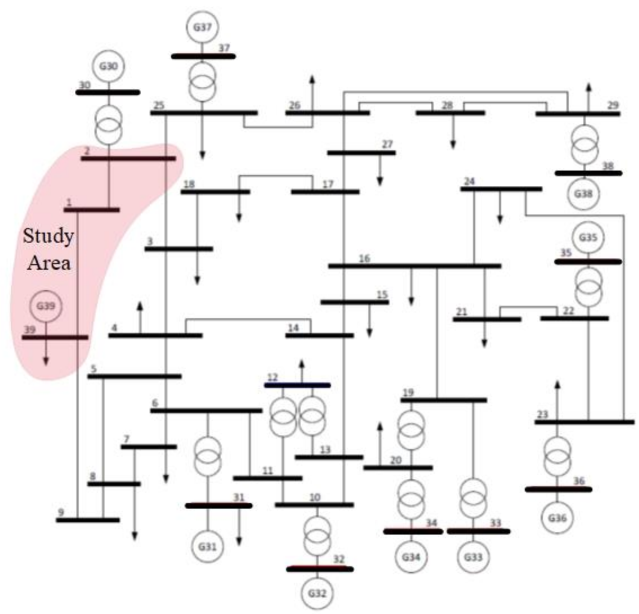

Fig. 2. Selected study area of the IEEE 39 bus system

The admittance matrix of the external area is reduced to a 2x2 matrix, as there are only two boundary buses (bus 2 and 39). The admittance matrix of the reduced network and its real components are given by equations (22) and (23), respectively. The values in the both equations (22) and (23) have per-unitized on 345 kV and 100 MVA base.

$$Y_{eq} = \begin{bmatrix} 12.92 - 41.23i & 1.36 + 5.21i \\ 1.36 + 5.21i & 1.52 - 10.29i \end{bmatrix} \tag{22}$$

$$Real(Y_{eq}) = \begin{bmatrix} 12.92 & 1.36 \\ 1.36 & 1.52 \end{bmatrix} \tag{23}$$

The minimum realization of the admittance matrix of the network equivalent (Y_{eq}) is shown in Fig. 3. For simplicity, bus 2 and 39 are referred to as 'Node 1' and 'Node 2' in the figures.

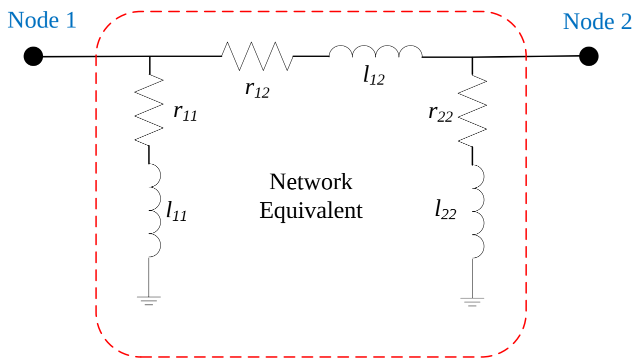

Fig. 3. Network of Y_{eq}, synthesized at power frequency

Note that, since real parts of off-diagonal elements of reduced admittance matrix are positive (23), the synthesized network has negative resistive elements in it.

Applying (19) on (22) yields,

$$Y' = \begin{bmatrix} Y_{eq_{1,1}} + \frac{Y_x^2}{Y_z} & Y_{eq_{1,2}} + \frac{Y_x Y_y}{Y_z} & Y_x \\ Y_{eq_{2,1}} + \frac{Y_x Y_y}{Y_z} & Y_{eq_{2,2}} + \frac{Y_y^2}{Y_z} & Y_y \\ Y_x & Y_y & Y_z \end{bmatrix} \tag{24}$$

General inequalities given in (20) and (21) are reduced to the following form for the selected example. These inequalities are needed to be satisfied to have a network with all positive resistive elements.

$$Real\left(Y_{eq_{1,2}} + \frac{Y_x Y_y}{Y_z}\right) \leq 0 \tag{25}$$

$$Real(Y_x) \leq 0 \tag{26}$$

$$Real(Y_y) \leq 0 \tag{27}$$

$$Real\left(Y_{eq_{1,1}} + \frac{Y_x^2}{Y_z} + Y_{eq_{1,2}} + \frac{Y_x Y_y}{Y_z} + Y_x\right) \geq 0 \tag{28}$$

$$Real\left(Y_{eq_{2,2}} + \frac{Y_y^2}{Y_z} + Y_{eq_{2,1}} + \frac{Y_x Y_y}{Y_z} + Y_y\right) \geq 0 \tag{29}$$

$$Real(Y_z + Y_x + Y_y) \geq 0 \tag{30}$$

Solution space of the above inequalities contain an infinite number of non-trivial solution and following solution from the solution space was arbitrarily selected. However, further constraints can be introduced to narrow down the solution space in order to obtain a more meaningful solution. For example, an additional constraints can be added to obtain a desired X/R ratio for equivalent branches or to match damping at a given frequency to better reflect the power system.

$$\begin{bmatrix} Y_x \\ Y_y \\ Y_z \end{bmatrix} = \begin{bmatrix} -2.87 - 4.73i \\ -0.75 - 2.37i \\ 5.45 - 7.93i \end{bmatrix} \tag{31}$$

Therefore, Y' can be reformulated using (24) and (31) as,

$$Y' = \begin{bmatrix} 9.76 - 40.83i & -0.06 + 5.05i & -2.87 - 4.73i \\ -0.06 + 5.05i & 0.91 - 10.51i & -0.75 - 2.37i \\ -2.87 - 4.73i & -0.75 - 2.37i & 5.45 - 7.93i \end{bmatrix} \tag{32}$$

As per (32), the new network equivalent has all positive resistive elements (see Fig. 4).

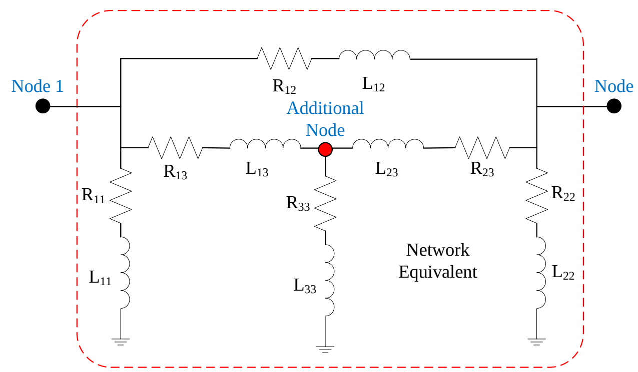

Fig. 4. Network of Y', synthesized on power frequency

A. Simulation Results

All three networks, namely, original network, reduced network (network with Y_{eq}) and proposed network (network with Y') were simulated using PSCAD/EMTDC with a time step of 10 µs. Voltage at bus 2 (node 1), bus 39 (node 2) and current in the branch between bus 39 and bus 1 were analyzed.

Fig. 5, Fig. 6 and Fig. 7 show the simulated voltage at bus 2, bus 39 and current in the branch between bus 39 and bus 1 for the original network (IEEE 39 bus system), respectively. The same parameters obtained for the reduced network (network equivalent depicted in Fig. 3) and the proposed network (network equivalent depicted in Fig. 4) are also presented in the same plots.

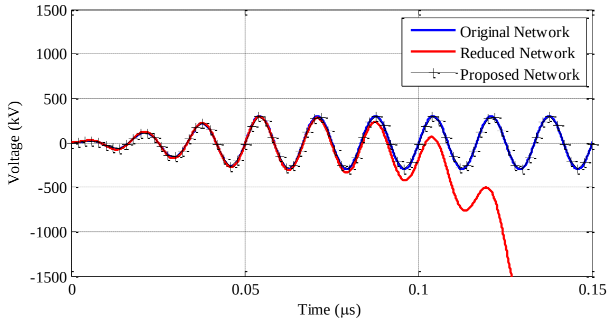

Fig. 5. Voltage at bus 2 for original, reduced and proposed networks

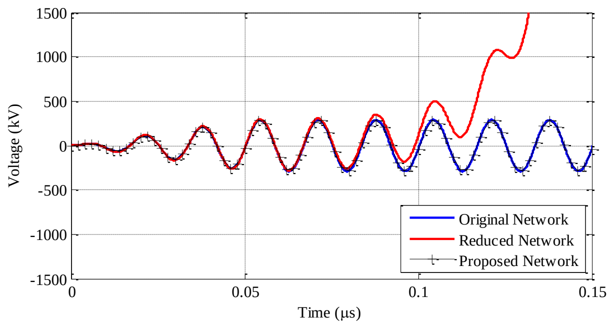

Fig. 6. Voltage at bus 39 for original, reduced and proposed networks

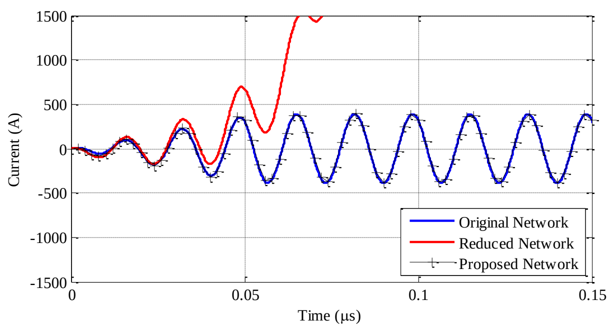

Fig. 7. Current between bus 39 and bus 1 measured at bus 39 for original, reduced and proposed networks

It is obvious that all three quantities from the reduced network indicate an unstable simulation. On the other hand, same quantities for the proposed network (network equivalent depicted in Fig. 4) are clearly stable.

The proposed network equivalent is initially benchmarked against power flow and short circuit current from the original network as shown in Table 1.

TABLE I
POWER FLOW AND SHORT CIRCUIT LEVEL COMPARISON

Network Element	Parameter	Original Network	Proposed Network
Branch (1-2)	Active Power (MW)	116.3	116.3
	Reactive Power (MW)	47.0	47.0
Branch (39-1)	Active Power (MW)	116.2	116.2
	Reactive Power (MW)	123.4	123.4
Bus 2	3-ph Short Circuit Current (kA)	8.7	8.7
Bus 39		5.8	5.8

EMT simulation results following a system disturbance (an ABC-G fault at bus 1) for the original network and the proposed network are compared in Fig. 8, Fig. 9 and Fig. 10. The results present the transients observed during clearing of the fault. Both networks were modeled in PSCAD/EMTDC and run with a simulation time-step of 10 μs.

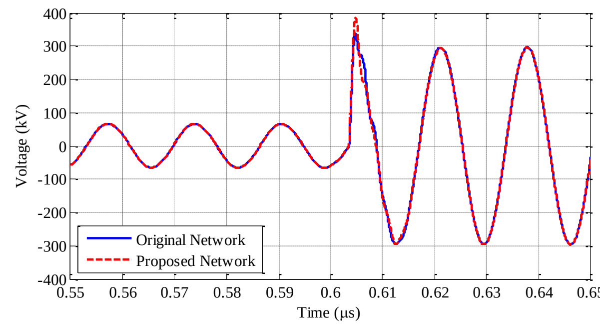

Fig. 8. Voltage at bus 1 for original and proposed networks during clearing of an ABC-G fault

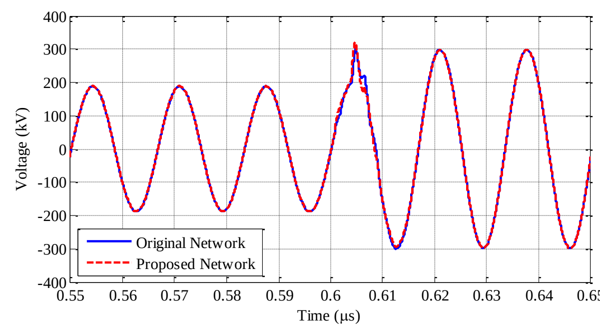

Fig. 9. Voltage at bus 2 for original and proposed networks during clearing of an ABC-G fault

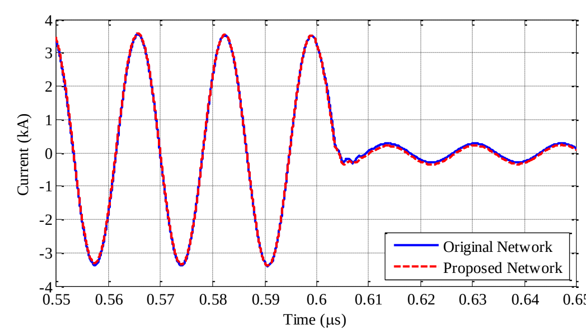

Fig. 10. Current between bus 1 and 2 measure at bus 1 for original and proposed networks during clearing of an ABC-G fault

Fig. 8, Fig. 9 and Fig. 10 show that the transient voltages and currents (observed following the disturbance) obtained using the proposed network agree with those from the original network. This confirms that the network reduction using the proposed technique eliminates the instabilities of the simulation while maintaining the accuracy.

B. Discussion

The active power, reactive power and short circuit current remain unchanged following the elimination of negative resistances using the proposed technique. Moreover, the EMT simulation results with the network reduced using the proposed technique agreed well with those from the original network. This demonstrates that the proposed technique does not diminish the accuracy of the network equivalent.

As per the definitions given in section II, the admittance matrix of the initial external area is positive-real, representing a stable system. Due to existence of negative resistances, the admittance matrix of the reduced network (using Kron reduction) is not positive-real, and therefore, does not have guaranteed stability. In this example, the reduced network with negative resistances ended up yielding unstable EMT simulations. It is important to notice that while networks with negative resistances are prone to instabilities, their presence does not always cause EMT simulations to be unstable. However, re-synthesizing the reduced network to have a positive-real admittance matrix assures the EMT simulation stability, as indicated by the above presented results.

V. Conclusions

This paper has investigated the effects of having negative resistances in network equivalents in EMT simulation and proposed a novel technique to mitigate them. The proposed technique in this paper efficiently eliminates all negative resistances in the network equivalent, and therefore the associated instabilities. The main advantage of this method is that all negative resistances in the network equivalent are eliminated by a single iteration. Rigorous mathematical formulation of the proposed technique was presented and its applicability was validated using a benchmark example case. The results presented indicated that the proposed method is more effective than other existing methods in eliminating the negative resistances introduced by the network reduction process.

VI. Acknowledgment

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VII. References

[1] Martinez-Velasco, Juan A., ed. Transient analysis of power systems: solution techniques, tools and applications. John Wiley & Sons, 2014.

[2] G. Kron, *Tensor Analysis of Networks*, Wiley, 1939.

[3] F. Dorfler and F. Bullo, "Kron Reduction of Graphs with Applications to Electrical Networks," in IEEE Transactions on Circuits and Systems I: Regular Papers, vol. 60, no. 1, pp. 150-163, Jan. 2013.

[4] Manitoba Hydro International (Sep. 2016) Webinar: Transient Recovery Voltage (TRV) Study [Online]. Available: https://hvdc.ca/uploads/knowledge_base/breaker_trv_studies.pdf?t=1466194850

[5] H. Dong, H. Weng, F. Han, H. Li, F. Xu, S. Liu, Z. Xu, "Reduction and modelling method of large-scale alternating current/direct current power systems for electromagnetic transient simulation," in IET Generation, Transmission & Distribution, vol. 8, no. 10, pp. 1667-1676, Oct. 2014.

[6] C. J. Xia, G. K. Liang, H. Y. Huang, "An Investigation of the Negative Resistance in Network Reduction and Electromagnetic Transient Modelling", Advanced Materials Research, Vols. 1008-1009, pp. 480-484, 2014.

[7] J. Rommes and W. H. A. Schilders, "Efficient Methods for Large Resistor Networks," in IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems, vol. 29, no. 1, pp. 28-39, Jan. 2010.

[8] Amir Mehdi Mousavi. "Frequency-Domain Equivalents for Passive Networks" Ph.D. diss., University of Toronto, 1999.

[9] R. Pintelon et al, "Parametric identification of transfer functions in the frequency domain-a survey", IEEE Trans. Automatic Control, vol. 39, no. 11, Nov. 1994, pp. 2245-60.

[10] M. A. Pai, *Energy function analysis for power system stability*. Kluwer academic publishers, Boston/Dordrecht/London, 1989.