Calculation of Lightning Electromagnetic Fields Above a Lossy Ground for Statistical Appraisal of Induced Overvoltages

F. Tossani, F. Napolitano, A. Borghetti, C. A. Nucci

***Abstract*—The waveform of lightning-induced overvoltages in overhead lines is significantly affected by the ground conductivity, due to the influence that ground losses have on the lightning-originated electromagnetic field. The rigorous evaluation of the lightning electromagnetic field for the case of a lossy ground can be accomplished by solving the Sommerfeld integrals, which generally require very demanding computational resources. The paper proposes a method for the efficient evaluation of the Sommerfeld integral that uses a trapezoidal scheme in which the nodes are equidistant in logarithmic scale. The method has been included in the LIOV-EMTP code for allowing the accurate calculation of lightning induced voltages on overhead lines above a lossy ground. The effects of the finite conductivity of the ground on the lightning induced overvoltages waveforms are discussed. In particular, the results from the proposed method are compared with those obtained by using the Cooray-Rubinstein formula, which is an established approximation. Further, the peak-amplitude probability-distribution of the induced voltages obtained by using the two approaches is presented. It is found that the accurate calculation is needed only for lines with a high insulation level and above a poor conducting ground.**

***Index Terms*—Cooray-Rubinstein formula, power distribution lines, lightning induced overvoltages, Monte Carlo method, Sommerfeld integrals.**

I. Introduction

THE accurate assessment of the response of distribution networks against a lightning electromagnetic pulse (LEMP) requires models of a certain complexity, as they must allow for the proper representation of real power distribution systems and of the electromagnetic environment. The statistical assessment of the lightning performance of such complex systems can be carried out by the application of the Monte Carlo method [1] in which a great number of time domain simulations is required.

In order to reduce the computational burden, one of the most common assumptions is the use of approximated formulas for the calculation of the LEMP in presence of lossy ground.

While the vertical component of the electric field and the azimuthal component of the magnetic field are slightly affected by the ground conductivity, the horizontal component of the electric field is significantly influenced by the presence of a lossy ground [2].

The rigorous calculation of the electromagnetic field radiated by a vertical antenna over a lossy ground needs the solution of the so-called Sommerfeld integrals [3]. A method for the numerical evaluation of the electromagnetic field has been proposed in [4] and it is used in [5] to assess the accuracy of the Cooray-Rubinstein (CR) formula [2], [6], [7], which is an established approximated approach for the evaluation of the effect of finite ground conductivity on the radial component of the return stroke electric field. In [5], the comparison is limited to electromagnetic field waveforms, i.e. no analysis is presented concerning the induced overvoltages.

This paper proposes a method for the evaluation of the radial electric field based on the numerical solution of the Sommerfeld integral. To handle the oscillatory and rapidly decaying behavior of the integrand function, the numerical solution is performed by using a trapezoidal scheme in which the nodes are equidistant in logarithmic scale.

The proposed procedure has been included in the LIOV–EMTP-RV code [8], [9], which allows the calculation of lightning induced overvoltages on power lines by implementing a finite-difference time-domain (FDTD) solution method of the Agrawal *et al.* [10] field-to-multiconductor line coupling model. This has enabled us to compare the induced overvoltages calculated by using the proposed solution of the Sommerfeld integral and those obtained by using the CR formula for different positions along the line and different ground conductivity values. The comparison with the FEM model described in [11] is also provided.

To check the validity of the CR formula from a statistical perspective, the same comparison has been accomplished concerning the indirect lightning performance of an overhead distribution line calculated by using the rigorous solution and the approximated one. The results are presented for different ground conductivity values. The procedure for the lightning performance appraisal is based on a Monte Carlo method that includes the possibility to use the Heidler function for representing the waveform of the lightning current at the channel base [12].

The structure of the paper is the following. Section II describes the calculation method, with particular reference to the numerical evaluation of the Sommerfeld integral and the

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relevant inverse Fourier transform. Section III illustrates the time domain waveforms of horizontal electric fields and induced overvoltages calculated by using the numerical solution of the Sommerfeld integral and the CR formula. Section IV is devoted to the comparisons between the statistical assessments of the indirect lightning performance of an overhead line obtained by using the two approaches. Section V concludes the paper.

II. Calculation Method

The rigorous formulation of the electromagnetic field radiated by an oscillating Hertzian dipole over a finite conductivity ground is due to Sommerfeld [3].

For the calculation of lightning-induced overvoltages according to the Agrawal model, the component of the electric field along the line direction at different distances r from the channel is needed.

The radial electric field at a point distant r from the channel, at height z above the ground, can be seen as the sum of two components:

$$E_r(r,z,j\omega) = E_{r,p}(r,z,j\omega) + E_{r,\sigma}(r,z,j\omega) \tag{1}$$

where $E_{r,p}$ is the radial electric field in case of perfectly conducting ground and $E_{r,\sigma}$ is the component that accounts for the finite conductivity of the ground σ_g.

By assuming an infinitely long straight lightning channel, the lossy ground contribution $E_{r,\sigma}$ to the radial electric field is given by [13]

$$E_{r,\sigma}(r,z,j\omega) = -\frac{I(j\omega)}{j\omega 2\pi\varepsilon_0}\int_0^\infty \lambda^2 J_1(\lambda r) e^{-z\sqrt{\lambda^2-k^2}} G(\lambda)\, Q(\lambda) d\lambda \tag{2}$$

where ω is the angular frequency, j the imaginary unit, $I(j\omega)$ is the channel base current, J_1 denotes the Bessel function of the first kind of order one,

$$G(\lambda) = \frac{k^2\sqrt{\lambda^2 - k_1^2}}{k_1^2\sqrt{\lambda^2-k^2} + k^2\sqrt{\lambda^2-k_1^2}} \tag{3}$$

in which $k^2 = \omega^2\mu_0\varepsilon_0$ and $k_1^2 = \omega^2\varepsilon_0\varepsilon_{rg}\mu_0 + j\omega\mu_0\sigma_g$ are the wave numbers in air and lossy ground, respectively, and

$$Q(\lambda) = \frac{1}{j\omega / v + \sqrt{\lambda^2 - k^2} + 1/\alpha} \tag{4}$$

where v is the return stroke speed, α is the return stroke model attenuation constant which is infinite in case of TL model.

The integrand in (2) decays exponentially as λ goes to infinite. Quadrature formulas are obtained by variable transformations that allow the application of the trapezoidal rule to infinite and semi-infinite integrals [14]. These procedures have been adopted to calculate Sommerfeld integral tails as described in e.g., [15], [16].

In this paper, an efficient quadrature is obtained by setting the nodes of the trapezoidal integration equally spaced in logarithmic scale. Through the change of integration variable $\lambda' = \lambda \cdot r$, the first zero of the Bessel function is found at $\lambda' = 3.8317$, beyond which the function oscillates. To preserve the accuracy when the oscillations of the integrand are significant, we use 5000 nodes for λ' from 10^{-2} to 10^0 and 20000 nodes for λ' from 10^0 to 10^4. Thanks to the change of integration variable, the number of nodes and their position along the integration domain can be left unaltered for different distances r.

The ideal ground components of the LEMP are calculated directly in time domain by using the analytical formulation presented in [17] by assuming that the lightning return stroke current propagates along the channel according to the transmission line (TL) model. This analytical formulation, which is implemented in the LIOV–EMTP-RV code, allows for the representation of a generic waveform of the lightning current at the channel base. We assume the return-stroke propagation speed is equal to 150 m/µs.

The electric field in time domain is obtained by the inverse Fourier transformation (IFT) of (2), which needs to be evaluated at different frequencies. To improve the computational performance of the IFT procedure, we reasonably disregard the frequency content of the lightning current above a few MHz.

For the representation of the channel base current waveform we use the Heidler function [18] which in time domain is given by

$$\begin{aligned} i(t) &= \frac{I_0}{\eta}\frac{(t/\tau_1)^N}{1+(t/\tau_1)^N}\exp(-t/\tau_2) \\ \eta &= \exp\left[\left(-\frac{\tau_1}{\tau_2}\right)\left(\frac{\tau_2 N}{\tau_1}\right)^{1/N}\right] \end{aligned} \tag{5}$$

The exponential approximation of the Heidler function described in [19] is adopted for the implementation in the frequency domain.

Fig. 3 shows the magnitude of two different return stroke current waveforms representative of a typical first stroke and subsequent stroke, respectively. The first stroke is represented by a single Heidler function with the following parameters: I_0 = 29.3 kA, τ_1 = 1.44 µs, τ_2 = 91.8 µs, n = 2. The subsequent stroke is given by the sum of two Heidler functions with the following parameters: I_{01}=10.7 kA, τ_{11} = 0.25 µs, τ_{21}=2.5 µs, n_1 = 2, and I_{02} = 6.5 kA, τ_{12} = 2.1 µs, τ_{22} = 230 µs, n_2 = 2. In both the cases, the magnitude of the current decreases quickly as frequency increases.

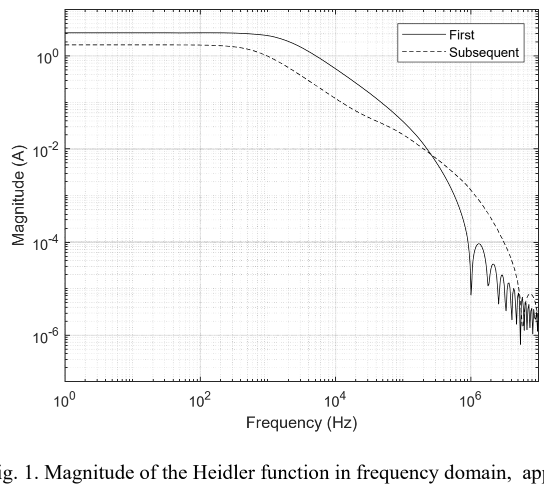

Fig. 1. Magnitude of the Heidler function in frequency domain, approximated with exponentials [19]. Comparison between a typical first and subsequent stroke.

For the IFT of the Sommerfeld component, we use the trapezoidal rule with 500 nodes equally spaced in logarithmic scale between 1 Hz and 10^7 Hz. We have verified that this number of nodes is adequate to obtain correct results up to 8 µs.

When a statistical population of channel base currents needs to be dealt with, e.g. when a Monte Carlo procedure is performed, the time to peak of the lightning current may deviate significantly from its median value and assume values up to some tens of µs. For these events, the simulation time must be increased significantly and 500 samples in frequency domain are in general not enough for preserving the IFT accuracy. To overcome this issue, a Hermite interpolation in the frequency domain is performed by adding 20 additional samples between two frequencies at which the Sommerfeld integral is evaluated.

III. Time domain results

Fig. 2 compares the radial electric field due to a subsequent stroke calculated by using the proposed procedure with the results provided by the FEM model described in [11]. Different distances from the lightning channel, namely $r = 51$, 502.5, 1001, 2001 m are considered. The observation points are 10 m above the soil, which has conductivity $\sigma_g = 1$ mS/m. These comparisons, along with others not reported here, confirm the good agreement between the FEM results and those provided by the proposed procedure, for observation points up to 4 km from the lightning channel.

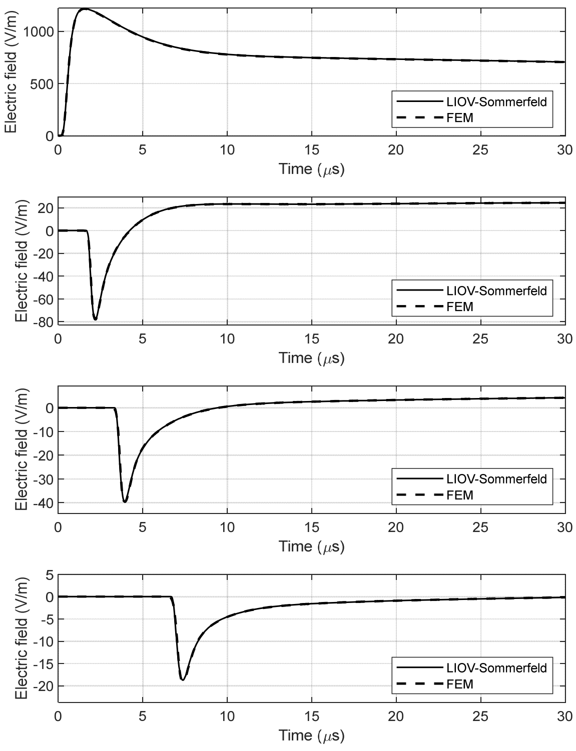

Fig. 2. Radial electric field due to a subsequent stroke calculated at $r = 51$, 502.5, 1001, 2001 m from the lightning channel. $\sigma_g = 1$ mS/m.

The following comparisons show the induced-overvoltages in a single-conductor, 1-km long overhead line calculated by using three different approaches for evaluating the radial component of the electric field, which for convenience we shall denote as follows: LIOV-Sommerfeld, which include the numerical solution of the Sommerfeld integral described in Section II, the FEM model, LIOV-CR, which adopts the CR formula (included in the time domain calculation procedure by using the piecewise linear transformation technique described in [20], [21]). The line conductor is located at a height of 10 m from the soil and its diameter measures 1 cm. The line terminations are matched.

In Fig. 3 and Fig. 4, the overvoltage induced in the middle of the line and at line terminations by a subsequent stroke located at 50 m from the line center are shown for a ground conductivity $\sigma_g = 1$ mS/m. Similar comparison between the results obtained by using the FEM and the LIOV-EMTP-RV code with the CR formula can be found in [11].

Fig. 5 shows the distribution of the overvoltage peak amplitude along the line. The use of the CR formula leads to a small overestimation of the induced voltage peak and a somewhat different waveform at the terminations with respect to the other two approaches.

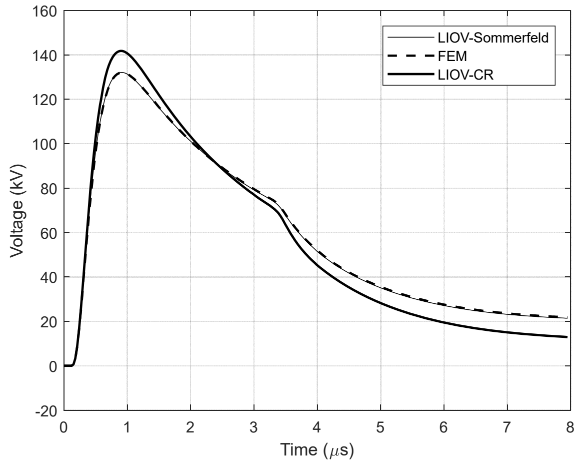

Fig. 3. Overvoltage in the middle of the line due to a subsequent stroke. $\sigma_g = 1$ mS/m.

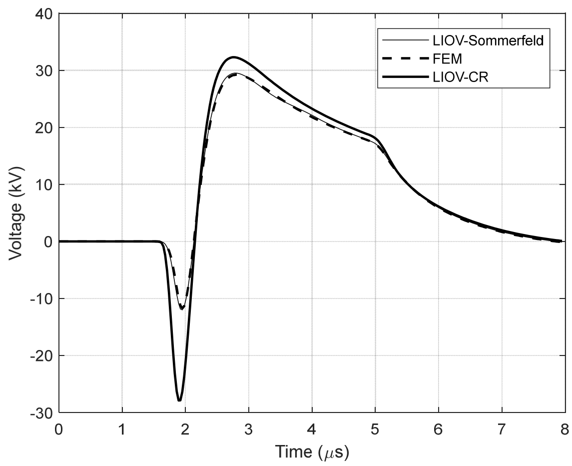

Fig. 4. Overvoltage at line terminations due to a subsequent stroke. $\sigma_g = 1$ mS/m.

The same comparisons are reported in Fig. 6, Fig. 7, and Fig. 8 for a low ground conductivity $\sigma_g = 0.1$ mS/m. In this case, the overvoltage peak is significantly underestimated by using the CR formula and the waveform appears more distorted with respect to the case of σ_g = 1 mS/m.

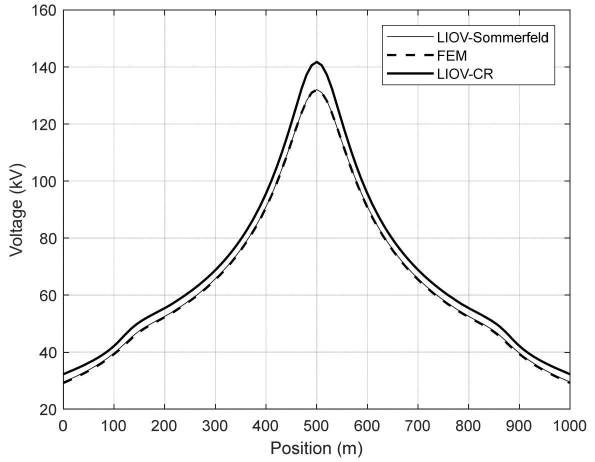

Fig. 5. Overvoltage peak amplitude along the line due to a subsequent stroke. σ_g = 1 mS/m.

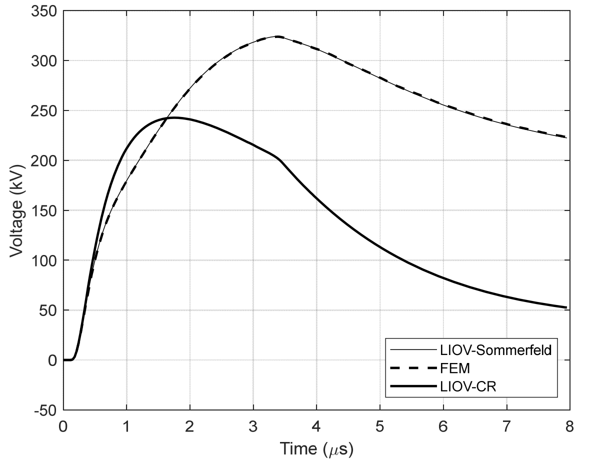

Fig. 6. Overvoltage in the middle of the line due to a subsequent stroke. σ_g = 0.1 mS/m.

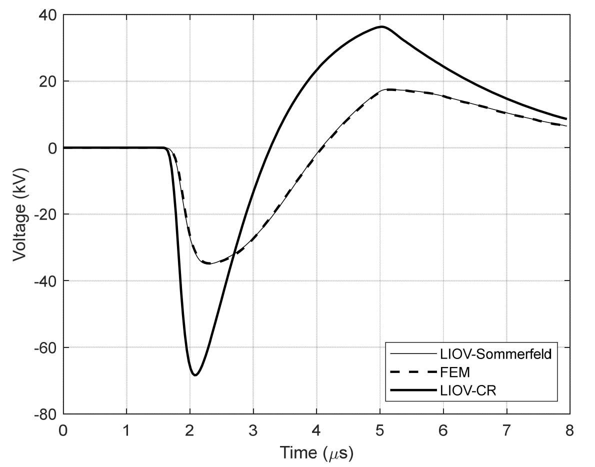

Fig. 7. Overvoltage at line terminations due to a subsequent stroke. σ_g = 0.1 mS/m.

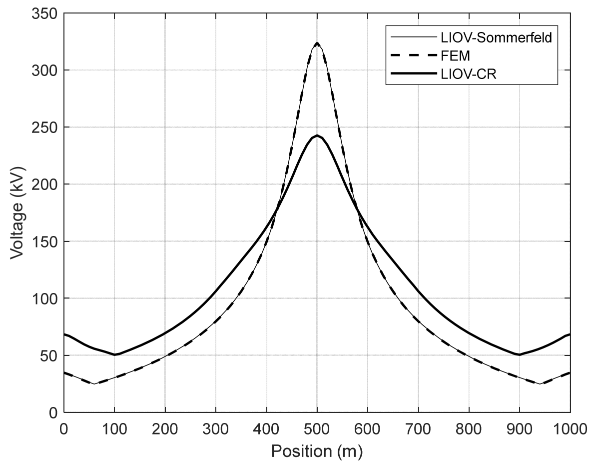

Fig. 8. Overvoltage peak amplitude along the line due to a subsequent stroke. σ_g = 0.1 mS/m.

The computational time required for the solution of the Sommerfeld integral is noticeably higher with respect to the CR approach. For a single time-domain simulation, e.g., to obtain the curves in 5, the LIOV-Sommerfeld and the LIOV-CR solution require 63 s and 1.4 s, respectively, using a PC equipped with a 2.6 GHz Intel i7-6700HQ CPU running 64-bit Windows 10.

IV. Lightning Performance Assessment

The lightning performance analysis is focused on a single-conductor 2-km long overhead line. The appraisal of the lightning performance is carried out by using a Monte Carlo method. The results of this paper are obtained by randomly generating n_{tot} = 20000 lightning events which are uniformly distributed within a striking area A of 4 km^2. The area contains all the lightning events inducing voltages larger than 75 kV. Each lightning event is classified either as a direct strike to the line or as an indirect strike by using the electro-geometric model as in [22].

The induced voltages are calculated by means of the LIOV–EMTP-RV code. The simulations are repeated to account for the effect of the ground conductivity using both the Sommerfeld integral and the CR formula. The expected annual numbers of events F_p able to induce overvoltages with peak amplitude larger than a given value V is:

$$F_p = \frac{n}{n_{tot}} A\, N_g \tag{6}$$

where n is the number of indirect events generating, in any point of the line, overvoltages larger than V and N_g is the annual ground flash density, assumed equal to 1 flash/km^2/yr.

Besides the stroke location, each lightning event is characterized by four more parameters in order to define the Heidler function that represents the lightning channel base current waveform: peak I_p, front time t_f, maximum front steepness S_m, and wave-tail time to half value t_h.

The Monte Carlo method requires the generation of the parameter values according to the relevant multivariate distribution. To this purpose, the log-normal probability distributions provided by Berger and Garbagnati in [23] and reported in Table I and Table II are adopted, where T_{cr} is the current time-to-crest. Further details on the procedure for the generation of the 20000 Heidler functions parameters are provided in [12].

TABLE I. STATISTICAL PARAMETERS OF THE LOG-NORMAL DISTRIBUTIONS FOR NEGATIVE DOWNWARD FIRST STROKES [23]

Parameter	Median value	Standard deviation of the parameter logarithm (base 10)
I_p	30 kA	0.26
T_{cr}	5.5 μs	0.31
S_m	12 kA/μs	0.26
t_h	75 μs	0.26

TABLE II. CORRELATION COEFFICIENTS BETWEEN PARAMETERS [23]

Parameter	I_p	T_{cr}	S_m	t_h
T_{cr}	0.37	1		
S_m	0.36	-0.21	1	
t_h	0.56	0.33	0.1	1

Fig. 9 shows the results of the statistical procedure based on the Monte Carlo method, i.e., the curves of the expected number per year of lightning events inducing overvoltages with peak amplitude greater than the value V in abscissa, for three different ground conductivity values. The curves show increasing discrepancies between the results obtained by the Sommerfeld integral and the CR formula as the value of V increases, especially for the lowest ground conductivities. For $\sigma_g = 0.1$ mS/m the CR formula underestimates the lightning performance. However, for typical insulation levels of medium voltage distribution lines, the CR formula provides results that are very close to the ones given by the numerical solution of the Sommerfeld integral.

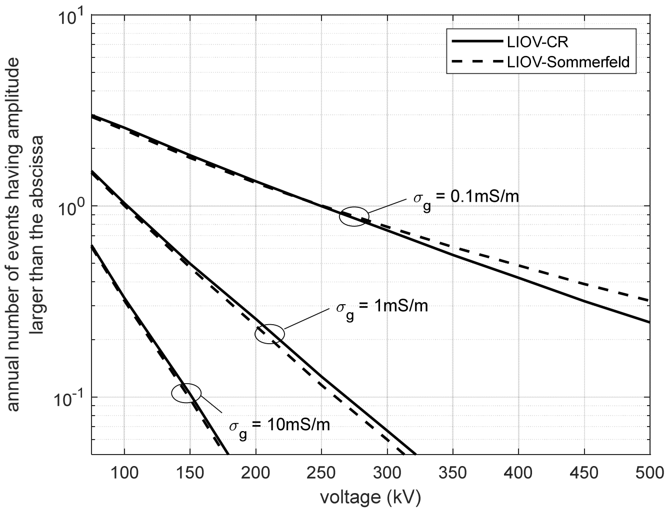

Fig. 9. Lightning performance of the line for different ground conductivities.

CONCLUSION

This paper proposes a method for the appraisal of the electric field generated by indirect lightning return strokes above a lossy ground, based on the numerical evaluation of the Sommerfeld integrals. The relevant procedure, which uses a trapezoidal rule with logarithmically spaced nodes, is both computationally efficient and accurate.

Such a procedure has been included in the time domain solution of the field-to-line coupling equations implemented in the LIOV-EMTP-RV code via inverse Fourier transform. The results have been compared with those obtained by using a FEM model and with those achieved by using the Cooray-Rubinstein formula. The time domain simulations of the induced overvoltages in an overhead line show that the FEM model gives results in close agreement with the ones obtained by using the numerical solution of the Sommerfeld integral, whilst the adoption of the Cooray-Rubinstein formula leads to attenuations and distortions for very low ground conductivity values.

In the indirect lightning performance of a typical distribution line, the differences between the overvoltages peak amplitudes calculated by the various approaches become negligible. Despite the slight overestimation of the voltage peak amplitudes, the Cooray-Rubinstein still represents a practical and consistent approach in statistical assessment of the indirect lightning performance, due to its much lower computational cost.

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