Efficient Implementation of Frequency Dependent Network Equivalents using State Space Models of Cascaded Sub-Circuits

Meysam Ahmadi, Shengtao Fan, Huanfeng Zhao and Aniruddha M. Gole

***Abstract*—One method to form a guaranteed passive frequency dependent equivalent (FDNE) of a large network is to use Brune's realization approach to synthesize a network of cascaded sub-circuits, given a measured or plotted impedance frequency response characteristic. In this work, each of the cascaded sub-circuits obtained from Brune's synthesis is mathematically represented by a set of Differential Algebraic Equations (DAEs), which are then combined together to form a state equation system of the full network. The resulting state space model can then directly be implemented on an Electromagnetic Transient (EMT) solver.**

***Keywords*: Brune's Realization Method, DAEs, FDNE, Fitting, Passive Network, State Space Model.**

I. Introduction

FREQUENCY Dependent Equivalent Networks (FDNEs) are used to speed up simulations by modeling parts of the network of less interest by simplified equivalents whilst maintaining the accuracy of the response from the driving point [1]. FDNEs were initially fitted by several branches of a fixed structure (e.g., several parallel arms with series R, L and C) which can be easily implemented in Electromagnetic Transient (EMT) programs [2-7]. However this representation was unable to capture any arbitrary passive system's response accurately. Later, vector fitting was introduced to fit the scan data by means of rational functions [8-10]. These vector-fitted rational functions can be readily implemented in an EMT program using exponential terms and solved using recursive convolution [11] or directly modeled as RLC branches [12]. However, additional steps must be applied to vector fitting to give a passive realization, and this is not always straightforward.

A new network realization-based approach was introduced by the authors [13] for modeling FDNEs utilizing Brune's realization method [14]. Brune's synthesis is a method to directly create a guaranteed passive formulation consisting of several sub-circuits. It can be applied to either tabulated or mathematically computed frequency response data. Vector fitting and other methods use a more complex algorithm for passivity enforcement. Brune's approach, by repeating some standard steps, results in a network which is a cascade connection of several sub-circuits (with positive R, L and C components and/or ideal transformers) with well-defined topology for each sub-circuit. To implement high order FDNEs using such networks, it is more efficient to make a state space model of the network, rather than modelling the resulting Brune's equivalent circuit directly into the solver. This is because the voltages or currents at internal nodes of the Brune equivalent are of no interest, as only the terminal behavior is important from the external circuit's point of view. On the other hand, a standard EMT solver would solve for these unnecessary internal quantities. Also, presence of several ideal transformers in the Brune's realization method, makes the modeling difficult in EMT solver. Alternatively, the state variable form allows a more compact and numerically efficient solution. This can be done by modeling each of the sub-circuits in the form of a differential algebraic system and then connecting them together to yield a state space model of the whole network. This paper proposes a procedure to generate state space model of a large network made up of several cascade sub-circuits whose configurations and thus DAEs are already known. The state space representation can then be easily implemented in an EMT solver. Here we have used PSCAD/EMTDC.

In addition to the FDNE application, the proposed state space generation of cascaded sub-circuits can be used in several applications. These include transmission cascaded pi sections [15], cross-bonded cables with cascaded segments [16], finite difference time domain models of a multiple segment cable transmission system [16] and nonuniform transmission line systems [17].

The layout of the remainder of the paper is as follows. Section II explains how DAEs of Brune's sub-circuits are used to represent state-space model of a multi-port FDNE. Section III discusses the conversion of the DAE model into a conventional state-space model and its EMT implementation. Section IV shows simulation examples, and conclusions are presented in Section V.

II. FDNE Implementation

Tellegen [20] extended Brune's single-port synthesis approach to realize a passive multi-port network consisting of N sub-circuits as shown in Fig. 1.

In the proposed approach, differential-algebraic equations and input and output relations are derived for each sub-circuit

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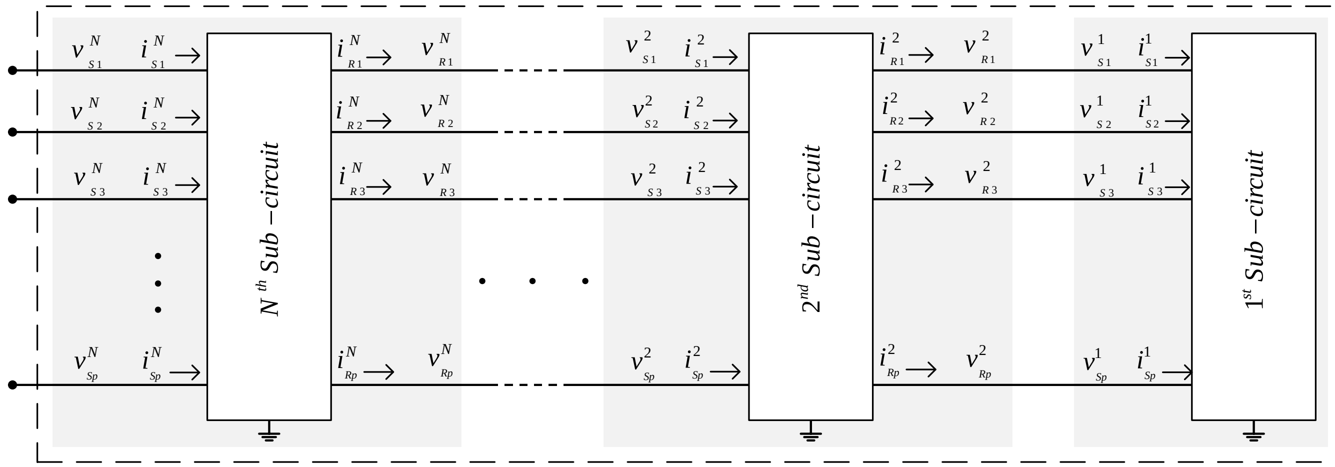

Fig. 1. Cascade connection of *N* sub-circuits

and then combined to make the state-space of the entire network.

A. State Space Generating Procedure

Tellegen's procedure, similar to Brune's single port realization method, ends with the very last step producing a pure resistance [15]. In our model, we start with this resistance as our first sub-circuit as in Fig. 1. In the multi-port case, this becomes a resistance matrix; with terminal voltage relation of $v^1_S=R\times i^1_S$. Where v^1_S is $p\times 1$ vector of the voltages, i^1_S is $p\times 1$ vector of the currents and R is a $p\times p$ resistance matrix.

Differential equations and output relations of all other sub-circuits can be derived as differential algebraic equations (DAEs) [18] given in (1) where matrix M could be singular. In (1), vector v and i are the terminal voltages and currents and x is the vector of state variable of the sub-circuits. A dot above x indicates its derivative. If M is non-singular, it can be inverted and the equations would be in classical state space form. The details of generation of DAEs of the sub-circuits will be discussed in section IV.

In the DAE representation, currents of the terminals of each sub-circuit are taken as output and the voltages are inputs. This selection, enables the model to be implemented by an EMT solver as an admittance function.

For convenience, the left hand side of a sub-circuit is referred to as the "sending end" and the right side as the "receiving end" as shown in Fig. 1. In equations (1.a) and (1.b), x^k is the vector of state variables, i_S^k and v_S^k are the sending end and i_R^k and v_R^k are the receiving end currents and voltages vectors of the k^{th} sub-circuit shown in Fig 1.

$$M^k \begin{pmatrix} \dot{x}^k \\ i^k \end{pmatrix} = N^k \begin{pmatrix} x^k \\ v^k \end{pmatrix} \quad \Rightarrow$$

$$\begin{matrix} a) \\ b) \end{matrix} \begin{pmatrix} M_{11}^k & | & M_{12}^k \\ -- & -|- & -- \\ M_{21}^k & | & M_{22}^k \end{pmatrix} \begin{pmatrix} \dot{x}^k \\ i_S^k \\ - \\ i_R^k \end{pmatrix} = \begin{pmatrix} N_{11}^k & | & N_{12}^k \\ -- & -|- & -- \\ N_{21}^k & | & N_{22}^k \end{pmatrix} \begin{pmatrix} x^k \\ v_S^k \\ - \\ v_R^k \end{pmatrix} \qquad (1)$$

As seen in Fig. 1, the sending end of the k^{th} sub-circuit directly connects to the receiving end of the $(k–1)^{th}$ sub-circuit. Hence, the *sending end* voltages and currents of the k^{th} sub-circuit are the same as the *receiving end* voltages and currents of the $(k–1)^{th}$ sub-circuit. Therefore, by cancelling all of these internal node voltages and currents, only the sending terminal of the N^{th} sub-circuit will remain which is the driving point terminal of the whole circuit.

Starting from the 2nd sub-circuit, the receiving end currents and voltages (i^2_R and v^2_R which are equal to i^1_S and v^1_S) must be canceled in the formulation.

We know that the first sub-circuit is a resistive network, as it is the last step in Brune's synthesis. Also, in (1), vectors $[\dot{x}^2 \; i_S^2]^T$ and $[x^2 \; v_S^2]^T$ are renamed as to $\dot{X}_{is}^2$ and X_{vs}^2 respectively to make the equations more compact. Therefore, to cancel i^2_R and v^2_R, the two equations of (2) can be written as:

$$\begin{cases} v_S^1 = R i_S^1 \quad \Rightarrow \quad v_R^2 = R i_R^2 \\ M_{21}^2 \dot{X}_{is}^2 + M_{22}^2 i_R^2 = N_{21}^2 X_{vs}^2 + N_{22}^2 v_R^2 \end{cases} \qquad (2)$$

Then i^2_R and v^2_R will be given by (3);

$$\begin{pmatrix} i_R^2 \\ v_R^2 \end{pmatrix} = \begin{pmatrix} -R & I \\ M_{22}^2 & -N_{22}^2 \end{pmatrix}^{-1} \begin{pmatrix} 0 & 0 \\ -M_{21}^2 & N_{21}^2 \end{pmatrix} \begin{pmatrix} \dot{X}_{is}^2 \\ X_{vs}^2 \end{pmatrix} \qquad (3)$$

This means i^2_R and v^2_R can be written in the form of (4).

$$\begin{cases} i_R^2 = K_{11} \dot{X}_{is}^2 + K_{12} X_{vs}^2 \\ v_R^2 = K_{21} \dot{X}_{is}^2 + K_{22} X_{vs}^2 \end{cases} \qquad (4)$$

In which the matrix K is given by (5);

$$\begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix} = \begin{pmatrix} -R & I \\ M_{22}^2 & -N_{22}^2 \end{pmatrix}^{-1} \begin{pmatrix} 0 & 0 \\ M_{21}^2 & -N_{21}^2 \end{pmatrix} \qquad (5)$$

Substituting the relations for i_R^2 and v_R^2 from (4) in (1.a) gives:

$$\begin{aligned} & M_{11}^2 \dot{X}_{is}^2 + M_{12}^2 (K_{11} \dot{X}_{is}^2 + K_{12} X_{vs}^2) = \\ & N_{11}^2 X_{vs}^2 + N_{12}^2 (K_{21} \dot{X}_{is}^2 + K_{22} X_{vs}^2) \end{aligned} \qquad (6)$$

Equation (6) can be simplified to (7):

$$\dot{X}_{is}^2 = P^{-1} Q X_{vs}^2 \qquad (7)$$

In which

$$\begin{cases} P = M_{11}^2 + M_{12}^2 K_{11} - N_{12}^2 K_{21} \\ Q = N_{11}^2 + N_{12}^2 K_{22} - M_{12}^2 K_{12} \end{cases} \qquad (8)$$

In (7), the state space equations can be found by unwrapping the compact form of $\dot{X}_{is}^2$ and X_{vs}^2 into (9)

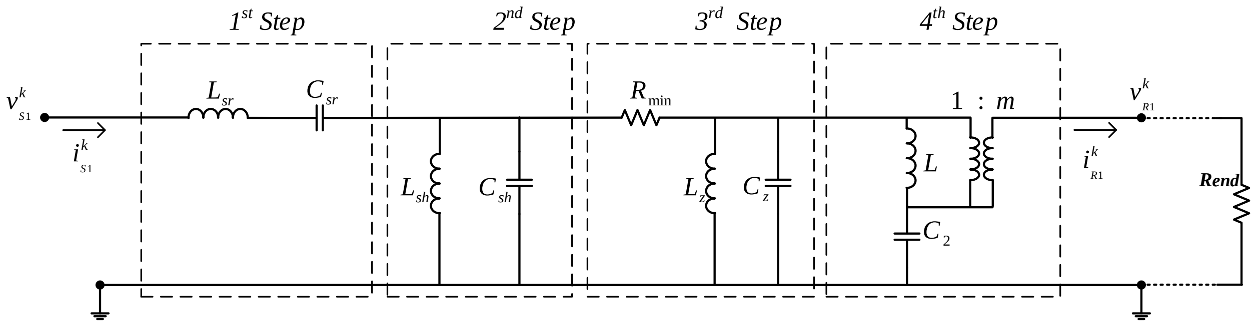

Fig. 2. Sub-circuits configuration in Brune's realization

$$\begin{pmatrix}\dot{x}^2\\ i_S^2\end{pmatrix}=P^{-1}Q\begin{pmatrix}x^2\\ v_S^2\end{pmatrix}\quad\Rightarrow\quad\begin{pmatrix}\dot{x}^2\\ i_S^2\end{pmatrix}=\begin{pmatrix}A^2 & B^2\\ C^2 & D^2\end{pmatrix}\begin{pmatrix}x^2\\ v_S^2\end{pmatrix}\tag{9}$$

Obviously (9) only gives the state space and output equations of combination of the 2nd and 1st sub-circuits together. For adding the 3rd and subsequent sub-circuits, a similar procedure as shown below is repeated. The resulting matrix equations are slightly different because we no longer have the condition that the first element was a resistor.

The receiving end currents and voltages of the third sub-circuit and later (i_R^k and v_R^k which are equal to i_S^{k-1} and v_S^{k-1}) must be canceled.

Assume that the state space form of all the *k-1* sub-circuits together is given by (10). x_o^{k-1} is a vector, including all the state variables of the previous *k-1* sub-circuits.

$$\begin{matrix}a)\\ b)\end{matrix}\quad\begin{pmatrix}\dot{x}_o^{k-1}\\ i_S^{k-1}\end{pmatrix}=\begin{pmatrix}A^{k-1} & B^{k-1}\\ C^{k-1} & D^{k-1}\end{pmatrix}\begin{pmatrix}x_o^{k-1}\\ v_S^{k-1}\end{pmatrix}\tag{10}$$

Then, (10.b) and (1.b) are used in (11) to add the k^{th} sub-circuit (*k=3, 4, 5….*) to the previous sub-circuits;

$$\begin{cases} i_S^{k-1}=C^{k-1}x_p^{k-1}+D^{k-1}v_S^{k-1}\\ \Rightarrow i_R^k=C^{k-1}x_p^{k-1}+D^{k-1}v_R^k\\ M_{21}^k\dot{X}_{is}^k+M_{22}^k i_R^k=N_{21}^k X_{vs}^k+N_{22}^k v_R^k\end{cases}\tag{11}$$

Solving (11) gives the i^k_R and v^k_R as shown in (12).

$$\begin{cases} i_R^k=T_{11}\dot{X}_{is}^k+T_{12}X_{vs}^k+T_{13}x_o^{k-1}\\ v_R^k=T_{21}\dot{X}_{is}^k+T_{22}X_{vs}^k+T_{23}x_o^{k-1}\end{cases}\tag{12}$$

Where the matrix T is given in (13),

$$T=\begin{pmatrix}I & D^{k-1}\\ M_{22}^k & -N_{22}^k\end{pmatrix}^{-1}\begin{pmatrix}0 & 0 & C^{k-1}\\ -M_{21}^k & N_{21}^k & 0\end{pmatrix}\tag{13}$$

Then after substituting i_R^k and v_R^k from (12) in (1.a):

$$\begin{aligned}M_{11}^k\dot{X}_{is}^k+M_{12}^k(T_{11}\dot{X}_{is}^k+T_{12}X_{vs}^k+T_{13}x_o^{k-1})=\\ N_{11}^kX_{vs}^k+N_{12}^k(T_{21}\dot{X}_{is}^k+T_{22}X_{vs}^k+T_{23}x_o^{k-1})\end{aligned}\tag{14}$$

Equation (14) can be simplified as in (15).

$$\begin{aligned}&\dot{X}_{is}^k=W^{-1}ZX_{vs}^k+W^{-1}Gx_o^{k-1}\\ &Where\begin{cases}W=M_{11}^k+M_{12}^kT_{11}-N_{12}^kT_{21}\\ Z=N_{11}^k+N_{12}^kT_{22}-M_{12}^kT_{12}\\ G=N_{12}^kT_{13}-M_{12}^kT_{23}\end{cases}\end{aligned}\tag{15}$$

Equation (15) only includes the "derivatives" of state variables of the k^{th} sub-circuit. The equations resulting from the k^{th} sub-circuit must be recursively combined with derivatives of those from the first *k-1* sub-circuits. Substituting v_R^k into (10.a), and considering (15) gives (16):

$$\begin{aligned}&\dot{x}_o^{k-1}=A^{k-1}x_o^{k-1}+B^{k-1}v_R^k\quad\Rightarrow\\ &\dot{x}_o^{k-1}=A^{k-1}x_o^{k-1}+B^{k-1}\left(T_{21}\dot{X}_{is}^k+T_{22}X_{vs}^k+T_{23}x_o^{k-1}\right)\\ &\Rightarrow\ \dot{x}_o^{k-1}=\left(A^{k-1}+B^{k-1}T_{21}W^{-1}G+B^{k-1}T_{23}\right)x_o^{k-1}+\\ &\qquad\left(B^{k-1}T_{21}W^{-1}Z+B^{k-1}T_{22}\right)X_{vs}^k\end{aligned}\tag{16}$$

Then the whole state space equation of the *k* sub-circuits together becomes (17);

$$\begin{aligned}&\begin{bmatrix}\dot{x}_o^{k-1}\\ \dot{X}_{is}^k\end{bmatrix}=\begin{bmatrix}E & F\\ W^{-1}G & W^{-1}Z\end{bmatrix}\begin{bmatrix}x_o^{k-1}\\ X_{vs}^k\end{bmatrix}\\ &Where\begin{cases}E=A^{k-1}+B^{k-1}T_{21}W^{-1}G+B^{k-1}T_{23}\\ F=B^{k-1}T_{21}W^{-1}Z+B^{k-1}T_{22}\end{cases}\end{aligned}\tag{17}$$

To see the classical state space form all the *k* sub-circuits, (17) can be unwrapped into (18).

$$\begin{aligned}&\begin{bmatrix}\dot{x}_o^{k-1}\\ --\\ \dot{x}^k\\ i_S^k\end{bmatrix}=\begin{pmatrix}E & | & F\\ -- & -|- & --\\ W^{-1}G & | & W^{-1}Z\end{pmatrix}\begin{bmatrix}x_o^{k-1}\\ --\\ x^k\\ v_S^k\end{bmatrix}\\ &\Rightarrow\begin{bmatrix}\dot{x}_o^{k-1}\\ \dot{x}^k\\ --\\ i_S^k\end{bmatrix}=\begin{pmatrix}A^k & | & B^k\\ -- & -|- & --\\ C^k & | & D^k\end{pmatrix}\begin{bmatrix}x_o^{k-1}\\ x^k\\ --\\ v_S^k\end{bmatrix}\end{aligned}\tag{18}$$

The process of removing internal nodes and recursively growing the state space matrices continues to *k=N* include all of the *N* sub-circuits to form A^N, B^N, C^N and D^N in (19) where x_o^N is the vector of state variable of the whole network.

$$\begin{bmatrix}\dot{x}_o^N\\ --\\ i_S^N\end{bmatrix}=\begin{pmatrix}A^N & | & B^N\\ -- & -|- & --\\ C^N & | & D^N\end{pmatrix}\begin{bmatrix}x_o^N\\ --\\ v_S^N\end{bmatrix}\tag{19}$$

B. Comments on Brune's Realization Method

Brune's method includes repeating cycles, each with four steps to generate a network that fits the tabulated impedance function. The network in each Brune cycle is as shown in Fig. 2. Some of the elements in Fig. 2 could turn out to be zero or infinity. At the end of the realization procedure, only the resistive network R_{end} is left.

In addition, Brune's method inherently preserves the passivity of the realized network, as all R, L and C component values are non-negative numbers. Brune's synthesis can realize a passive network for ***any positive real function*** [13].

The final realized network is made up of a cascade connection of several sub-circuits with known configuration of the type shown in Fig. 2, which is terminated in a resistive load considered as the first sub-circuit.

The ideal transformer in the circuit makes it difficult to obtain the standard state space form of this special circuit in a straightforward way. However, obtaining the DAEs for each sub-circuit is straightforward as will be shown next.

C. Generation of DAEs for Sub-Circuits

To create the state space equations, a variety of approaches are available e.g. modified nodal analysis (MNA) [18] or the conventional tree and co-tree method. In this work, the equations are derived using a slightly modified tree and co-tree method applied to the sub-circuit in Fig. 2. Due to the ideal transformer in the circuit, there will be a coupling between the output current equations and derivative of the state variables which forces the DAEs to be in form of (20) in which inductor currents and capacitor voltages are taken as the state variables (x), terminal voltages (v_S) are the inputs and terminal currents (i_S) are the outputs. The full matrices on M and N are given in the appendix. The appendix explains why matrix M is singular.

$$[M]\begin{bmatrix}\dot{x}\\ y\end{bmatrix}=[N]\begin{bmatrix}x\\ u\end{bmatrix} \tag{20}$$

III. Implementation on an EMT Solver

The generated state space model of the network can be readily transferred to any EMT solver by doing discrete time integration with a method such as the trapezoidal rule.

In this work, the FDNE module in PSCAD is used which itself accepts state space *ABCD* matrices data via an input file. Nevertheless, this FDNE module requires the state matrix A to be in diagonal form. However, the diagonalization of A can be done via a similarity transformation if needed [15] which is explained below.

Considering $Y(s)$ is the admittance of the network using state space form in (21), the matrix A can be converted into diagonal form Λ with the help of the eigenvector matrix T.

$$\begin{aligned} Y(s) &= C(sI-A)^{-1}B+D \\ \Rightarrow Y(s) &= CT(sI-\Lambda)^{-1}T^{-1}B+D \end{aligned} \tag{21}$$

If matrix T is complex, a further step is needed to make the state space matrices real. This can be done by defining residue matrices R_s as follows;

$$\begin{aligned} &R_s = C_x(:,s).B_x(s,:) \quad s=1,2,3,\dots n \\ &\textit{Where}\ \ C_x = CT \ \textit{and}\ B_x = T^{-1}B \end{aligned} \tag{22}$$

In (22), matrices R_s are formed by the multiplication of the s^{th} column of C_x by the s^{th} row of B_x. There will be n matrices, where n is the number of states.

The new state space matrices after transformation will be of the form given in (23) where, λ_s are the eigenvalues of the original state matrix A and identity matrix of *[I]* are $p\times p$ with p is the number of ports in the final circuit.

$$A_t=\begin{bmatrix}\lambda_1[I] & & & & & \\ & \lambda_2[I] & & & & \\ & & \ddots & & & \\ & & & \lambda_s[I] & & \\ & & & & \ddots & \\ & & & & & \lambda_n[I]\end{bmatrix}$$

$$B_t=\begin{bmatrix}[I]_1\\ [I]_2\\ \vdots\\ [I]_s\\ \vdots\\ [I]_n\end{bmatrix} \qquad \begin{aligned} C_t &= [R_1\ R_2\ \cdots R_s\ \cdots R_n] \\ D_t &= D \end{aligned} \tag{23}$$

IV. Simulation Results

To verify the proposed algorithm for the state space model generation and the EMT implementation, two case studies are presented in the section.

A. Case 1: A Simple Two Port Circuit

To validate the functionality of the method, a two port FDNE of the RLC network shown in Fig. 3 is scanned and then realized using Brune's approach. Then the network is converted into state-space model using proposed method. In this case, The FDNE is made out of 6 sub-circuits.

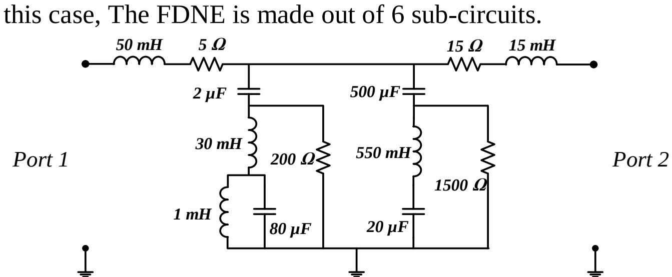

Fig. 3 The two port network under study

Fig. 4 shows the step response of the original network and the proposed state space model. 1kV and 2kV Step voltages are applied to port 1 and 2 at time *0.01sec* and *0.1sec* respectively. The graph shows excellent agreement between the responses.

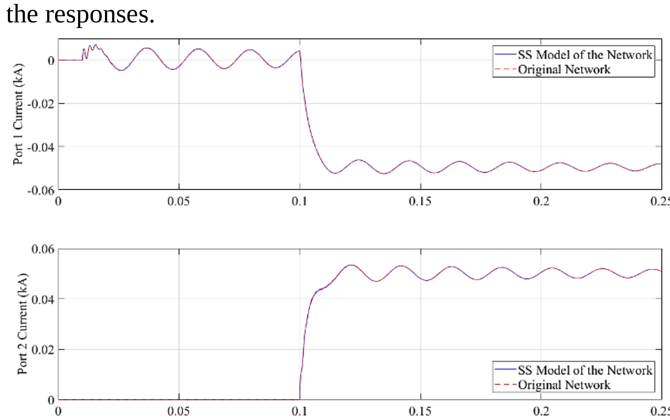

Fig. 4 Step response of the original network compared to the proposed state space model

B. Case 2: A More Realistic Network

The network in Fig. 5 shows a 22kV source connected to a remote load with a 100km long cable and a filter bank is also present. FDNE of the network from the Bus 2 point of view is modeled using Brune's method which resulted in 14 sib-circuits of the type shown in Fig. 2.

To test this case, two line-to-ground faults (0.05 ohms) are applied on Bus 2 and at the middle of the short line at $t = 0.02s$ and $t=0.20s$ respectively. Duration of each fault is *0.06s*.

Fig. 6 shows the source current before and after applying each faults comparing the original network and the reduced model using proposed state space model. Excellent agreement is achieved between the simulation of the original network and the reduced one in both transient and the steady state responses.

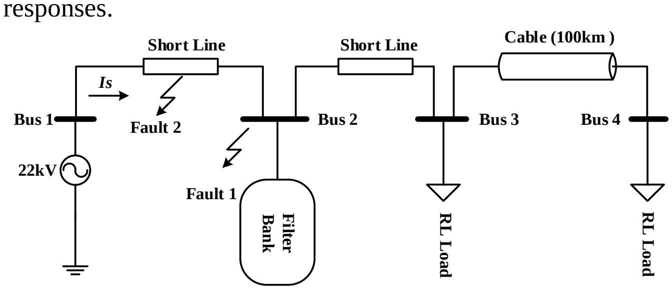

Fig. 5 The network, fault locations and the sending end current (Is). Data is in the appendix

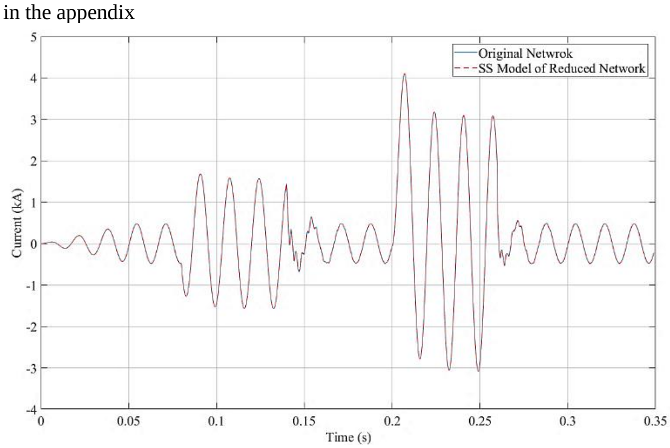

Fig. 6 Source current comparison during and after faults

V. Conclusion

In this work a new procedure is introduced to generate the state space model of a network composed of multiple cascade sub-circuits which are represented by DAEs. This procedure is a necessary step in implementing Brune's procedure for a multi-port Frequency Dependent Network Equivalent, and gives a guaranteed passive realization for the FDNE. The state space model of the network from the driving point is systematically generated for implementing in an EMT program. The driving point admittance of the network is modeled by the proposed state space approach. The procedure to implement the model in an EMT solver such as PSCAD/EMTDC is presented. The method is validated by some case studies and different scenarios in which the time domain responses obtained from the proposed state space model of the FDNE and the original unreduced networks are compared. Results show excellent agreement between the responses.

Brune's procedure could be terminated at any iteration with a partial realization and still yield a guaranteed passive network. The accuracy versus simulation time tradeoff is an area for future investigation.

VI. Appendix

Formation of matrix *M* is given in part A and example system data is provided in part B.

A. Matrix M and N of the Brune's Sub-Circuit

M and *N* matrices of the DAEs derived for the circuit in Fig. 2 are given in (A.1). The matrix *M* is singular due to the two linearly dependant rows containing only *1* and *m*. Therefore, it is not possible to directly derive the classical state space form of the sub-circuit.

B. Example System Data

Small T-line sections modelled as L-R circuits:
L=50 *mH*, R =5 Ω
RL Loads:
R = 125 Ω, L =180 *mH*
Filters: 1 *MVAR* equally distributed between 3^{rd}, $11^{th}/13^{th}$ double tuned and $24^{th}/26^{th}$ double tuned HP filters.
Cable:
Resistivity = 1.68e-8 Ω.m, *Relative Permittivity* = 4.1 and *Relative Permeability* = 1.

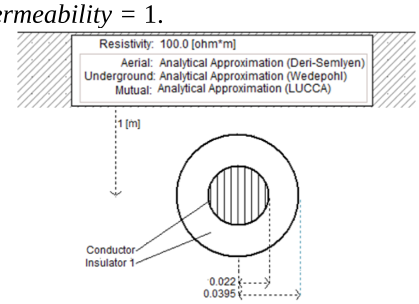

Fig. 7 Cross section of the cable buried 1 meter underground.

VII. References

[1] A. S. Morched, J. H. Ottevangers, and L. Marti, "Multi-port frequency dependent network equivalents for the emtp," *IEEE Transactions on Power Delivery*, vol. 8, pp. 1402–1412, July 1993.

[2] N. G. Hingorani and M. F. Burbery, "Simulation of ac system impedance in hvdc system studies," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-89, pp. 820 – 828, May 1970.

[3] A. Clerici and L. Marzio, "Coordinated use of TNA and digital computer for switching- surge studies: Transient equivalent of a complex network," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-89, pp. 1717–1726, Nov 1970.

[4] A. Morched and V. Brandwajn, "Transmission network equivalents for electromagnetic transients studies," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-102, no. 9, p. 29842994, 1983.

[5] V. Q. Do and M. M. Gavrilovic, "An iterative pole-removal method for synthesis of power system equivalent networks," *IEEE Power Engineering Review*, vol. PER-4, pp. 42–43, Aug 1984.

[6] N. R. Watson and J. Arrillaga, "Frequency-dependent ac system equivalents for harmonic studies and transient convertor simulation," *IEEE Transactions on Power Delivery*, vol. 3, pp. 1196–1203, Jul 1988.

[7] A. Morched, L. Marti, and J. Ottevangers, "A high frequency transformer model for the emtp," *IEEE Transactions on Power Delivery*, vol. 8, pp. 1615–1626, July 1993.

[8] B. Gustavsen and A. Semlyen, “Simulation of transmission line transients using vector fitting and modal decomposition,” *IEEE Transactions on Power Delivery*, vol. 13, pp. 605–614, April 1998.

[9] B. Gustavsen and A. Semlyen, “Rational approximation of frequency domain responses by vector fitting,” *IEEE Transactions on Power Delivery*, vol. 14, pp. 1052–1061, July 1999.

[10] A. Morched, B. Gustavsen, and M. Tartibi, “A universal model for accurate calculation of electromagnetic transients on overhead lines and underground cables,” *IEEE Transactions on Power Delivery*, vol. 14, pp. 1032–1038, July 1999.

[11] A. Semlyen and A. Dabuleanu, “Fast and accurate switching transient calculations on transmission lines with ground return using recursive convolutions,” *IEEE Transactions on Power Apparatus and Systems*, vol. 94, pp. 561–571, Mar 1975.

[12] A. C. S. Lima, B. Gustavsen, and A. B. Fernandes, “Inaccuracies in network realization of rational models due to finite precision of RLC branches,” in International Conference on Power Systems Transients (IPST07), June 2007.

[13] M. Ahmadi, A. M. Gole, “A New Approach to Model Frequency Dependent Network Equivalents in Transient Simulation Tools,” in International Conference on Power Systems Transients (IPST17), June 2017

[14] O. Brune, “Synthesis of a finite two-terminal network whose driving-point impedance is a prescribed function of frequency,” *Journal of Math and Physics*, vol. 10, pp. 191–236, 1931.

[15] B. Gustavsen and A. Semlyen “A Robust Approach for System Identification in the Frequency Domain,” *IEEE Trans. Power Delivery*, vol. 19, no. 3, pp. 1167–1173, July 2004

[16] K. K. M. A. Kariyawasam, S. Fan, A. M. Gole and H. M. J. S. P. De Silva, “Accurate validation of Electromagnetic Transient models of cascaded power transmission systems,” *2016 IEEE Power and Energy Society General Meeting (PESGM)*, Boston, MA, 2016, pp. 1-5.

[17] J. A. B. Faria and R. Araneo, “Computation, Properties, and Realizability of the Characteristic Immittance Matrices of Nonuniform Multiconductor Transmission Lines,” in *IEEE Transactions on Power Delivery*, vol. 33, no. 4, pp. 1885-1894, Aug. 2018.

[18] N. Banagaaya, G. Alì and W.H.A. Schilders, *Index-aware Model Order Reduction Methods - Applications to Differential-Algebraic Equations*, Atlantis Press, Paris, 2016, pp 1-2, [online] Available: https://doi.org/10.2991/978-94-6239-189-5

[19] L. Brančík, “Modified nodal analysis and state variable descriptions for MTLs systems solution in Matlab,” 2013 36th International Conference on Telecommunications and Signal Processing (TSP), Rome, 2013, pp. 354-357.

[20] B. D. H. Tellegen, “Synthesis of 2n-poles by networks containing the minimum number of elements,” *Journal of Math and Physics*, vol. 32, pp. 1–18, Apr. 1953.

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & C_{sh} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_z & 0 & 0 & 0 & 0 & 0 & 0 & m \\ 0 & 0 & 0 & C_2 & 0 & 0 & 0 & 0 & 0 & 1-m \\ 0 & R_{\min}C_{sh} & 0 & 0 & -L_{sr} & 0 & 0 & (1-m)L & -R_{\min} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & m & 0 & 0 \\ 0 & -C_{sh} & -C_z & 0 & 0 & 0 & 0 & 0 & 1 & -m \end{pmatrix} \begin{bmatrix} \dot{v}_{sr} \\ \dot{v}_{sh} \\ \dot{v}_z \\ \dot{v}_2 \\ \dot{i}_{sr} \\ \dot{i}_{sh} \\ \dot{i}_z \\ \dot{i}_L \\ - \\ i_{S1} \\ i_{R1} \end{bmatrix} =$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 & 1/C_{sr} & 0 & 0 & 0 & 0 & 0 \\ 0 & -1/R_{\min} & 1/R_{\min} & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1/R_{\min} & -1/R_{\min} & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -R_{\min} & 0 & 0 & -1 & 1 \\ 0 & 1/L_{sh} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/L_z & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/L & -1/L & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1/L & 0 & 0 & 0 & 0 & 0 & 1/L \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix} \begin{bmatrix} v_{sr} \\ v_{sh} \\ v_z \\ v_2 \\ i_{sr} \\ i_{sh} \\ i_z \\ i_L \\ - \\ v_{S1} \\ v_{R1} \end{bmatrix} \qquad \text{(A.1)}$$