On Combining Classical and Numerical Techniques for Extracting the Impedance and Admittance of Cables and Overhead Lines

H.M.J. De Silva, M. Shafieipour

Abstract—This paper discuses a general formulation of transmission line parameters for EMT-type programs by combining different computational techniques. The formulation of impedance and admittance matrices are described for various combination of overhead conductors, aerial or underground coaxial cables, pipe-type cables or even certain types of nonconventional cables, etc. The parameters of the transmission line system can be computed by combining different solution techniques to optimize computational speed and memory. Time domain simulations involving electromagnetic transients are carried out for an example consists of an overhead line, a pipetype cable and an aerial pipe cable in parallel. The formulation is also verified with alternative Method of Moment technique for an example case involving a non-conventional cable.

Keywords: Electromagnetic transient modelling, Impedance and admittance matrices, General formulation, Cables and overhead lines

I. INTRODUCTION

A Transmission line system in general can consist of combination of overhead lines, many types of underground/overhead cables mutually coupled together. Due to lack of availability of land, this is a common situation in urban areas and as a result, several overhead lines systems and/or cable systems share the same right of way. In recent years, there is an increasing necessity in studying induced voltages and currents on pipelines due to mutual coupling between overhead lines and pipelines. It is not uncommon to have many different types of cable systems in parallel as several European countries prefer the replacement of overhead lines with underground cables.

Reference [1] discusses the general cable parameter derivation for pipe-type cables, coaxial cables (aerial or underground) using traditional approach. The traditional method considers the skin effect, but neglects the proximity effect between cables [1]-[6]. The effect of proximity effect is said to be small and hence neglected in many Electro-Magnetic Transient (EMT) simulation studies.

The recent development of Method of Moment (MoM) parameter calculation technique [2]-[5],[16] can be used to formulate any combination of conductors. The MoM based methods consider the skin effect as well as the proximity

effect. For coaxial cables, pipe-type cables and also arbitrary shaped cables such as sector-shaped cables, per unit length parameters can be extracted using sub-conductor technique [14], MoM SVS-EFIE [2],[4] or MoM CIM-SO [3]. However for transmission line configurations with many conductors, the computation time and memory requirement can be significantly high [4]. For circular shapes such as coaxial cables or pipe-type cables, traditional approach [1],[6] or MoM-SO method [5],[16] can be used and they are computationally more efficient (in terms of computation time and also memory requirement) compared to the above techniques (e.g. MoM SVS-EFIE).

For a given combination of various cables and overhead lines, a significant saving on computational effort can be achieved by choosing appropriate computation method for each cable (or system of cables in close proximity) and integrating them to form a per-unit length parameters for the entire system. For example, if there is a sector-shaped cable in parallel with three coaxial cables, a complete solution via MoM SVS-EFIE (or sub-conductor technique) can be computationally expensive. Alternatively, the internal parameters of sector-shaped cable, coaxial cables can be extracted efficiently using MoM SVS-EFIE and traditional method respectively and then the full impedance and admittance matrices for entire system can be derived based on the general formulation discussed in section III.

Although there are many publications discussing about parameters derivation for overhead lines or cables, a description of general formulation combining different computational techniques with various combination of conductors/cables could not be found. The present paper discusses the formulation of transmission line parameters for various combinations of cables and can be considered as an extension of [1]. Note that overhead conductors can be also modelled as aerial bare cables [1].

In this paper, the general term "cable" refers to pipe-type cable, coaxial cable, overhead conductor or non-conventional cable. For a given transmission line configuration, the cables can be underground or aerial or combination of both. The proposed method generalizes and improves the Line / Cable Constant routines in EMT-type simulation tools.

II. REVIEW OF GENERAL FORMULATION

The reference [1] discusses modeling of pipe-type cable, coaxial cables and overhead lines in a general form. The perunit length series impedance matrix of a cable can be written

H.M.J. De Silva (e-mail: jeewantha@mhi.ca). and M. Shafieipour are with the Manitoba Hydro international, Canada (e-mail: mshafieipour@mhi.ca).

Paper submitted to the International Conference on Power Systems Transients (IPST2019) in Perpignan, France June 17-20, 2019.

as,

$$Z = Z_i + Z_{outer} \tag{1}$$

(a) For coaxial-cables or overhead conductor, the internal impedance (Z_i) can be written as (see (6) in [1]),

$$Z_i = Z_{int} \tag{2}$$

(b) For pipe-type cables with inner cables, Z_i is [1]

$$Z_i = Z_{int} + Z_p + Z_c \tag{3}$$

where Z_{int} , Zp and Zc are the internal impedance, pipe internal impedance, connection impedance between pipe inner and outer surfaces respectively [1]. The internal impedance matrix of pipe-type cable is a block diagonal matrix and if there are m cables inside the pipe, the Z_{int} can be written as [1],

$$Z_{int} = \begin{bmatrix} [Z_{int1}] & 0 & . & 0 & 0 \\ 0 & [Z_{int2}] & . & 0 & 0 \\ . & . & . & 0 & 0 \\ 0 & 0 & 0 & [Z_{intm}] & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
(4)

where Z_{inti} is the *i*th cable internal impedance inside pipetype cable. If the pipe is empty, only Z_c exits. The derivation of the internal impedances can be found in many publications and an improvement is discussed in Appendix.

The details of the earth return impedance (Z_{outer}) is described in [1] (see Z_0 in (7) and (32) of [1]). The earth return impedance can be computed using the [7] and [8] for overhead and underground cables respectively.

The more accurate earth return formulas can be found in [11],[12],[13]. The updated earth return impedance and admittance considers the earth permittivity value and enhances the accuracy at high frequencies. The numerical stability of the time domain simulation is improved. (E.g. Note that authors observed unstable time domain simulations in a Gas Insulated Substation (GIS) study case involving short aerial cables. The numerical stability problem was solved when more accurate earth return formulas were used).

The per-unit length shunt admittance (Y) is related to the potential coefficient matrix (P) as [1], [6],

$$Y = jwP^{-1} \tag{5}$$

where w is the angular frequency. The general form of potential coefficient matrix is,

$$P = P_{inti} + P_0 \tag{6}$$

(a) For coaxial-cables or overhead conductors

$$P_{inti} = P_i \tag{7}$$

(b) For pipe-type cables with inner cables, the P_{inti} matrix has the general form (see 42 in [1]),

$$P_{inti} = P_i + P_p + P_c \tag{8}$$

where P_i , P_p , P_c are the internal potential coefficient, pipe internal potential coefficient and potential coefficient matrix between pipe inner and outer surfaces. The pipe internal potential coefficient matrix (P_i) has the form (44 in [1]),

$$P_{it} = \begin{bmatrix} P_{i1} & 0 & . & 0 & 0 \\ 0 & [P_{i2}] & . & 0 & 0 \\ . & . & . & 0 & 0 \\ 0 & 0 & 0 & [P_{in}] & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
(9)

where P_{ii} is the potential coefficient matrix of i^{th} cable inside pipe-type cable. P_0 is the potential coefficient matrix for the cable system in air.

III. PROPOSED FURTHER GENERALIZATION

This paper extends the generalization discussed in section II so that,

(1) A transmission line system having various combinations of mutually coupled cables such as pipe-type cables, coaxial cables, overhead lines, arbitrary shaped cables can be modelled together

(2) A combination of different computational techniques can be used to calculate per-unit length parameters of the transmission line system

(3) Certain non-conventional cables can also be modelled.

The formulation is illustrated using an example as shown in figure 1 consisting of a three-phase overhead line, an underground pipe-type cable and an aerial pipe (hollow coaxial cable).

The per-unit length series impedance matrix of a transmission line system can be written as,

$$Z = Z_{internal} + Z_{outer} \tag{10}$$

where $Z_{internal}$ is the internal impedance of the transmission line system and has a block diagonal matrix,

$$Z_{internal} = \begin{bmatrix} [Z_1] & 0 & 0 & 0 & 0 \\ 0 & [Z_2] & 0 & 0 & 0 \\ 0 & 0 & [Z_3] & 0 & 0 \\ 0 & 0 & 0 & [Z_4] & 0 \\ 0 & 0 & 0 & 0 & [Z_5] \end{bmatrix}$$
(11)

where Z_1 , Z_2 and Z_3 are the overhead conductor internal impedances, Z_4 is the pipe-type cable internal impedance and Z_5 is the aerial pipe internal impedance. In general, Z_i is an $n \times n$ matrix representing the internal impedance of i^{th} cable, where n is the number of conductors of the cable. The overhead conductor is considered as a simple bare aerial cable (cable with inner conductor only). The Z_i can be computed using (2), (3) in section II.

The matrix Zouter represents the self and mutual impedances

between cables through air or earth. The earth return impedance matrix Z_{outer} has a following form,

$$Z_{outer} = \begin{bmatrix} [Z_{11}] & [Z_{12}] & [Z_{13}] & [Z_{14}] & [Z_{15}] \\ [Z_{21}] & [Z_{22}] & [Z_{23}] & [Z_{24}] & [Z_{25}] \\ [Z_{31}] & [Z_{32}] & [Z_{33}] & [Z_{34}] & [Z_{35}] \\ [Z_{41}] & [Z_{42}] & [Z_{43}] & [Z_{44}] & [Z_{45}] \\ [Z_{51}] & [Z_{52}] & [Z_{53}] & [Z_{54}] & [Z_{55}] \end{bmatrix}$$
(12)

If i^{th} cable is overhead and j^{th} cable is underground (or vice versa), then the formulas in [9] or [10] can be used to calculate mutual impedance between overhead lines and underground cables.

This formulation can be further extended to consider the certain types of non-conventional cables (see an example in figure 5). There can be pipe-type cables inside pipe conductor other than coaxial cables or bare conductors. (e.g. an internal pipe-type cable can be seen in figure 5 consisting of pipe (c4) and inner cables (c1,c2 and c3)).

The internal impedance (Z_i) of this pipe-type cable can be computed using (3). However, if the i^{th} inner cable is a pipe-type cable, Z_{inti} in (4) becomes,

$$Z_{inti} = Z_{int}' + Z_{p}' + Z_{c}'$$
(13)

where Z_{int}', Z'_p, Z_c' are the internal impedance, pipe internal impedance, connection impedance between pipe inner and outer surfaces of the internal pipe-type cable. Theoretically, this recursive formulation can be generalized to compute parameters of any combination of annulus shaped conductors.

One advantage of this generalization is the ability to combine parameters calculated from different computational methods. Alternatively, sub-internal impedance (Z_i) in (11) can be computed through MoM based methods [2]-[5],[16] or other techniques such as sub-conductor technique [14]. The selection of computational method for each cable (or combination of cables in close proximity) in (11) depends on the accuracy, computational effort and applicability.

The general potential coefficient matrix for the transmission line system is,

$$P = P_{internal} + P_0 \tag{14}$$

where $P_{internal}$ is the internal potential coefficient matrix of the transmission line system and has a block-diagonal format,

$$P_{internal}$$
(15)
=
$$\begin{bmatrix} [P_{int1}] & 0 & 0 & 0 & 0 \\ 0 & [P_{int2}] & 0 & 0 & 0 \\ 0 & 0 & [P_{int3}] & 0 & 0 \\ 0 & 0 & 0 & [P_{int4}] & 0 \\ 0 & 0 & 0 & 0 & [P_{int5}] \end{bmatrix}$$

where P_{inti} is the internal potential coefficient matrix of i^{th} cable.

The non-conventional cables (see Figure 5) can be approximately modelled as a pipe-type cable using (8). If the

 i^{th} inner cable is a pipe-type cable, P_{ii} in (9) can be written as,

$$P_{ii} = P'_{ii} + P_p' + P_c'$$
(16)

where P'_{ii}, P'_p, P'_c are the internal potential coefficient, pipe internal potential coefficient, and potential coefficient matrix between pipe inner and outer surfaces of the inner pipe-type cable. Similarly, alternative MoM based method can be used to calculate the sub-matrices of the internal potential coefficient matrix.

The P_o has the form (similar to (51) in [1]),

$$P_{0} = \begin{bmatrix} P_{011} & P_{012} & P_{013} & P_{014} & P_{015} \\ P_{021} & P_{022} & P_{023} & P_{024} & P_{025} \\ P_{031} & P_{032} & P_{033} & P_{034} & P_{035} \\ P_{041} & P_{042} & P_{043} & P_{044} & P_{045} \\ P_{051} & P_{052} & P_{053} & P_{054} & P_{055} \end{bmatrix}$$
(17)

If the *i*th cable is in air, P_{0ii} is the $n \times n$ self-potential coefficient matrix of *i*th cable. If both *i*th and *j*th cables are in air, the mutual potential coefficient matrix (P_{0ij}) can be calculated [1]. The self and mutual entries of potential coefficient matrix for underground cables and overhead lines can be found in [11],[12],[13] considering earth permittivity values.

A. Inclusion of shunt conductance

In a general transmission line system, there can be both overhead lines and cables. This section describes the inclusion of shunt conductance in such general system. The shunt conductance value (although very small) plays a significant role in improving the stability of the time domain simulation of HVDC transmission line and cables [17]. The shunt conductance value also defines the decay of the charge, once the line is de-energized.

1) Underground cables

For cables, the shunt conductance is a property of the insulation. The shunt conductance of each cable is defined in the internal potential coefficient matrix. The cable permittivity value becomes complex [11],

$$\varepsilon = \varepsilon' + j\varepsilon'' \tag{18}$$

where

$$\varepsilon'' = \varepsilon' tan\delta \tag{19}$$

 $tan\delta$ is the loss factor of the insulation and ε' is the relative permittivity of insulation material (e.g. for XLPE insulation, a typical value for ε' is 2.3) [11]

2) Overhead conductors

The shunt conductances of overhead conductors are added directly to the relevant diagonal elements of admittance matrix in (5). (i.e. corresponding to overhead conductors or aerial cables).

IV. APPLICATION EXAMPLES

In this section, the proposed general formulation is demonstrated using examples involving (a) a transmission line configuration having different cable types and overhead lines (b) a non-conventional cable. Time domain transient results for linear network impedance terminations are presented and the impedance parameters are compared with the MoM numerical technique [2] for case (b).

A. Example 1

The transmission line configuration in figure 1 consists of a three phase non-transposed line, an underground pipe-type cable and an aerial hollow pipe. The transmission line parameters such as series impedance matrix and shunt admittance matrix are calculated for a frequency range (0.5 Hz to 1 MHz) using the general formulation discussed in section III by MATLAB commercial software. The data is shown in table 1.



Figure 1. Transmission line configuration with overhead lines and cables

TABLE I
TRANSMISSION LINE DATA

Overhead transmission line	
Outer radius of conductor	0.0203454 m
Dc resistance of conductor	0.03206 ohm/km
Pipe-type cable	
Number of Inner Cables	3
Inner Insulator Outer Radius	0.025 m
Inner Ins. Relative Permittivity	2.3
Conductor Outer Radius	0.027 m
Conductor Resistivity	1.68e-07 Ωm
Outer Insulator Outer Radius	0.030 m
Outer Ins. Relative Permittivity	2.3
Aerial Pipe	
Conductor Inner Radius	0.02 m
Conductor Outer Radius	0.022 m
Conductor Resistivity	1.68e-08 Ωm
Other	
Ground resistivity	100 Ωm
Length of the line	5 km

The time domain simulation setup with linear network impedance terminations is shown in figure 2. The calculated parameters are entered through the external ZY data entry feature in the frequency dependent phase domain model (Universal Line Model) in PSCAD/EMTDC commercial software.

1) Time domain simulations

A 230 kV (L-L), 60 Hz voltage is applied to the overhead conductors and three inner conductors of pipe-type cable are energized with 11 kV (L-L), 60 Hz voltage source. A line to ground fault is applied to the C3 conductor at t = 0.1 s (with fault duration 0.05s) and the induced voltages on the underground pipe-type cable and the aerial pipe at receivingend are shown in figure 3 and 4 respectively. The solid curve refers the traditional approach for parameter calculation for all cables and the dotted curve represents the combined approach (MoM for the pipe-type cable internal parameters and traditional approach for the other cables). The two curves are in close agreements validating the proposed combined approach for parameter estimation.



Figure 2. Network configuration for the transient simulation



Figure 3. Induced voltage on underground pipe of the pipe-type cable



B. Example 2

The proposed formulation is general so that it can be applied to certain non-conventional configurations as shown in figure 5. This non-conventional cable consists of pipe-type cable having two hollow pipes and one inner pipe-type cable. The data for the cable can be found in table 2. The inner pipe-type cable has three solid conductors. The depth of the pipe is 1m below the ground surface. The per-unit length parameters are computed using the formulation discussed in section III.



Figure 5. Non-conventional pipe-type cable example

TABLE 2
CABLE DATA

Pipe-type cable	
Number of Inner Cables	3
Resistivity for all conductors	1.72e-08
Inner Ins. Relative Permittivity	3.0
Outer Ins. Relative Permittivity	2.3
Outer radius of pipe	0.02 m
Inner pipe-type cable	
Inner Ins. Relative Permittivity	3.0
Other	
Ground resistivity	100 Ωm
Length of the line	10 km

1) Comparison with Method of Moment technique(MoM) To validate the accuracy of the proposed general method, the parameters of the non-conventional cable are calculated by two methods, (a) traditional approach using proposed general formulation and (b) the MoM SVS-EFIE [2],[4] method. The magnitude of the first column of the impedance matrix is shown in figure 6. The impedance values from proposed formulation based on traditional method are in a close agreement with MoM parameter extraction method.



V. CONCLUSIONS

A general formulation of impedance and admittance matrices is proposed to model various combinations of different types of cables (including overhead lines) in parallel. The general formulation is described via time domain transient simulations involving such parallel combination and also a nonconventional type cable. The general parameter extraction for a non-conventional cable is also explained and verified with alternative MoM parameter extraction method. This general formulation can be used to eliminate limitations in existing transmission line and cable parameter algorithms in EMT-type software. As the parameters of each cable such as internal impedance can be computed using different techniques and then are combined together in formulating full matrices for the entire transmission system, a better numerically efficient solution can be achieved.

VI. APPENDICES

A. Discussion on pipe-type cable equations

The publication [1] discusses pipe-type equations in detail and hence not repeated in this paper (see also [13]). However after a kind discussion with Ametani, the following changes are made to improve the accuracy of the pipe-type cable equations. The more accurate pipe-type cable internal impedance (assuming a finite pipe thickness) can be expressed by modifying (34) and (37) in [1] as shown below,

$$Z_{pjk} = Q_{jk} + J_{jk} \tag{A1}$$

where,

$$J_{pjk} = 2\mu \sum_{n=1}^{\infty} \frac{C_n}{n(1+\mu_p) + x_1 K_{n-1}(x_1)/K_n(x_1)}$$
(A2)

The elements of sub-matrices of Z_c can be expressed as,

$$Z_{c1} = Z_{pi} + Z_{p0} + Z_{p3} - 2Z_{pm}$$
(A3)

$$Z_{c2} = Z_{p0} + Z_{p3} - Z_{pm} \tag{A4}$$

$$Z_{c3} = Z_{p0} + Z_{p3} \tag{A5}$$

Pipe inner impedance can be written as,

$$Z_1 =$$
(A6)

$$\frac{m_p \rho_p}{2\pi r_{p1}} \left[\frac{K_1(m_p r_{p2}) I_0(m_p r_{p1}) + K_0(m_p r_{p1}) I_1(m_p r_{p2})}{K_1(m_p r_{p1}) I_1(m_p r_{p2}) - K_1(m_p r_{p2}) I_1(m_p r_{p1})} \right]$$

The table 3 summarizes the above impedances

TABLE 3 IMPEDANCE COMPARISON

Impedance	Description	
Z _{pjk}	Elements of Z _p matrix	
Z _{pi}	Pipe inner impedance	ĺ
Z _{po}	Pipe outer impedance	ĺ
Z _{p3}	Outer pipe insulation impedance	Ĺ
Z _{pm}	Pipe mutual impedance	1'

B. Comparison with coaxial cable equations

A simple coaxial cable can be considered as a special case of pipe-type cable. If there is only one inner cable inside pipe at the center and the inner cable only consists of one conductor, then it can be shown that the pipe-type equations are identical to the coaxial cable equations.

The Q_{11} in (A1) becomes the inner insulation impedance and the J_{pjk} becomes zero as the eccentricity disappears. The inner, outer and mutual pipe impedances become corresponding to relevant sheath impedances. In matrix form, the equations are identical to (5) in [1] or section 2.3.4 in [6].

References

- A. Ametani, "A General Formulation of Impedance and Admittance of Cables", *IEEE Transactions on Power Apparatus and Systems, vol. PAS-*99, no. 3, pp. 902-910, May 1980.
- [2] M. Shafieipour, Z. Chen, A. Menshov, J. De Silva, V. Okhmatovski, "Efficiently computing the electrical parameters of cables with arbitrary cross-sections using the method-of-moments", *Electric Power Systems Research Volume 162, September 2018, Pages 37-49*
- [3] U.R. Patel, P. Triverio, "Skin effect modeling in conductors of arbitrary shape through a surface admittance operator and the contour integral method", *IEEETrans. Microw. Theory Tech.*, vol.64, pp. 2708–2717, *Sept. 2016.*
- [4] M. Shafieipour, J. De Silva, A. Kariyawasam, A. Menshov, V. Okhmatovski, "Fast computations of the electrical parameters of sectorshaped cables using single-source integral equation and 2D moment-

method discretization", Proceedings IPST-2017, Paper 1-6, Seoul, South Korea, June 2017.

- [5] H. Xue, A. Ametani, J. Mahseredjian and I. Kocar, "Computation of Overhead Line / Underground Cable Parameters with Improved MoM-So Method", PSCC-2018/Dublin, June 2018.
- [6] L. M. Wedepohl, D. J. Wilcox, "Transient analysis of underground power-transmission systems", Proc. IEE, vol. 120, pp. 253-260, 1973.
- [7] J. R. Carson, "Wave propagation in overhead wires with ground return", Bell Syst. Tech. J., vol. 5, pp. 539-554, 1926.
- [8] F. Pollaczek, "Über das Feld einer unendlich langen wechsel stromdurchflossenen Einfachleitung", pp. 339-360, 1926.
- [9] A. Ametani, T. Yoneda, Y. Baba and N. Nagaoka, "An Investigation of Earth-Return Impedance Between Overhead and Underground Conductors and Its Approximation", *IEEE Transactions on Electromagnetic Compatibility, vol. 51, Issue: 3, pp. 860 - 867, August* 2005.
- [10] Uribe F. A., "Calculating Mutual Ground Impedances Between Overhead and Buried Cables", *IEEE Transactions on Electromagnetic Compatibility, vol. 50, Issue: 1, pp. 198 - 203, February 2008.*
- [11] A. Ametani, "Further improvement of CABLE CONSTANTS and an investigation of cable problems", *EMTP News, vol. 1, no. 4, pp.4-14, Dec. 1988.*
- [12] A. Ametani, Y. Miyamoto and J. Mahseredjian, "Derivation of an earthreturn admittance of an overhead multi-conductors considering displacement currents", *IEEJ Trans. Power Energy*, vol.134, pp.936-940, 2014.
- [13] H. Xue, A. Ametani J. Mahseredjian and I. Kocar, "Generalized formulation of earth-return impedance/admittance and surge analysis on underground cables", *IEEE Trans. Power Delivery*, *TPWRD*.2018.2796089.
- [14] K.K.M.A. Kariyawasam, A. M. Gole, B. Kordi, H.M.J.S.P. De Silva, "Accurate Electromagnetic Transient Modelling of Sector-Shaped Cables", *Proceedings IPST-2011, Delft, The Netherlands, June 2011.*
- [15] A. Ametani, K. Fuse, "Approximate method for calculating the impedances of multiconductor cross section of arbitrary shapes", *Elect. Eng. Japan, vol. 112, no 2, pp. 117-123, 1992.*
- [16] U. R. Patel, P. Triverio, MoM-SO: "A complete method for computing the impedance of cable systems including skin, proximity, and ground return effects", *IEEE Transactions on Power Delivery*, vol. 30, pp. 2110-2118, 2015
- [17] H.M.J. De Silva, A.M. Gole and L.M. Wedepohl, "Accurate Electromagnetic Transient Simulations of HVDC Cables and Overhead Transmission Lines", *Proceedings IPST-2007, Lyon, France, June 2007.*