New Method of Moments Approach to Rigorous Analysis of Overhead and Buried Transmission Lines in the Presence of Air-Soil Interface

S. Zheng, M. Shafieipour, J. DeSilva, J. Nordstrom, V. Okhmatovski

***Abstract*—The paper extends a recently proposed novel Method of Moments approach to rigorous characterization of the power delivery overhead and buried transmission lines in the presence of soil-air interface. To introduce the method a study of the impact on the per-unit-length (p.u.l.) resistance (R) and inductance (L) of the two-conductor transmission lines (MTLs) from the presence of soil-dielectric interface typically encountered in the overhead and buried cables is demonstrated. The problem of the magneto-quasi-static analysis is formulated as the Surface-Volume-Surface Electric Field Integral Equation (SVS-EFIE) with multilayered medium Green's function. The latter is evaluated in the closed form using the finite-difference solution of the ordinary differential equation governing the spectrum of the Green's function. Numerical studies are performed for different elevations of the overhead line above the air-soil interface and depths of burial of the underground cables. It is demonstrated that while p.u.l. R and L values have notable impact from the presence of the soil half-space in the case of both low frequencies and high frequencies. Most pronounced impact is shown to occur from the air-soil interface at the high frequencies (e.g. frequencies of lightning discharge), especially when low depths of burial or overhead elevations are considered. The presented results are validated using COMSOL commercial Finite-Element-Method (FEM) extractor and PSCAD software.**

***Keywords*—Transmission lines, resistance, inductance, layered medium, Green's function, integral equations, method of moments**

I. Introduction

THE presence of the air-soil half-space may have a substantial or negligible impact on the p.u.l. characteristics of the MTLs depending on the location of the conductors with respect to the dielectric interface and frequency [1]. When the impact from the presence of the soil is significant the accuracy of the transient analysis of the MTLs [7] may be effected which may ultimately jeopardize the accuracy of the simulators based on such models [8].

To study the soil impact on the p.u.l. resistance and inductance in MTLs both approximate analytic formulas and rigorous numerical methods have been developed [2]-[6]. In this work we introduce into power delivery community a new rigorous Method of Moments (MoM) approach based on solution of a single source surface integral equation (SSIE) [9] for computation of the electromagnetic fields in the MTLs in the presence of a general multi-layered medium. The Green's function of the layered medium is evaluated in the closed-form using the finite difference evaluation of the Green's function spectrum followed by the analytic evaluation of the inverse Fourier transform from the resulting pole-residual approximation of the spectrum [10]. The Green's function samples are organized into a database [11], which allows to expediently compute the integrals of the Method of Moments (MoM) matrix elements using 3D interpolation of the Green's function values available in the database. The new methodology was initially introduced in [9]. In this work we demonstrate it's applicability for analysis of typical power delivery transmission lines and accuracy compared to the established methods such as FEM. The method provides closer match in Z-parameters to the FEM results [14] than analytic formulas implemented in PSCAD [8]. This shows that when accurate broadband R and L matrices are desired for performing transient simulations the proposed method can be utilized instead of the approximate analytic solutions.

To demonstrate the method the most commonly used case of the air-soil half-space is considered. The MoM computational framework based on the SVS-EFIE [9] is used for analysis of the two-conductor transmission line situated above and below the dielectric interface of the air-soil half-space. Numerically computed p.u.l. R and L values of the MTL are evaluated in the presence of the soil effect depending on the elevation of the line above or below the interface and frequency. Numerical results are presented for frequencies ranging from 60Hz to 100kHz. It is shown that both at low and high frequencies the impact on the p.u.l. R and L of the MTL is is substantial. It depends on both MTL elevation above or below the interface and the frequency. The numerical results generated using multi-layered medium formulation of the SVS-EFIE [9] are compared against the FEM solution of the COMSOL commercial solver [14] and PSCAD software utilizing classical analytic formulas for prediction of the impact from the air-soil interface.

II. SVS-EFIE Formulation in Layered Medium

The SVS-EFIE with respect to the unknown auxiliary surface current density J_z on the boundary of conductor is

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obtained through substitution of single-source field representation into classical Volume EFIE [15] and enforcing the latter at the observation points approaching the boundary of the conductor ∂S from inside its volume S [9]

$$-i\omega\mu_0 \oint\limits_{\partial S} G_\sigma(\boldsymbol{\rho},\boldsymbol{\rho}')\mathrm{J}_z(\boldsymbol{\rho}')\,d\boldsymbol{\rho}' + \sigma\omega^2\mu_0 \times \oint\limits_{\partial S}\left[\iint\limits_{S} G_\epsilon(\boldsymbol{\rho},\boldsymbol{\rho}')G_\sigma(\boldsymbol{\rho}',\boldsymbol{\rho}'')ds'\right]\mathrm{J}_z(\boldsymbol{\rho}'')d\boldsymbol{\rho}'' = V_{\text{p.u.l.}},\ \boldsymbol{\rho}\in\partial S. \quad (1)$$

where $G_\sigma(\boldsymbol{\rho},\boldsymbol{\rho}') = -(i/4)H_0^{(2)}(k_\sigma|\boldsymbol{\rho}-\boldsymbol{\rho}'|)$ is the Green's function of the conductor medium with wavenumber $k_\sigma = \sqrt{\omega\mu_0\sigma/2}(1-i)$, σ being the conductor bulk conductivity, and $H_0^{(2)}$ being the second kind Hankel function of zero-th order. The Green's function $G_\epsilon(\boldsymbol{\rho},\boldsymbol{\rho}')$ in (1) is the magnetic vector potential produced at the point $\boldsymbol{\rho}$ due to z-directed 2-D point source (filament of current) which is situated at the location $\boldsymbol{\rho}'$ in the layered medium.

The above SVS-EFIE (1) can be written in a concise operator form, as follows:

$$\mathcal{T}_\sigma^{\partial S,\partial S}\circ \mathrm{J}_z + \sigma\mathcal{T}_\epsilon^{\partial S,S}\circ\mathcal{T}_\sigma^{S,\partial S}\circ\mathrm{J}_z = V_{\text{p.u.l.}}. \quad (2)$$

The integral operators in (2) according to (1) are defined as

$$\mathcal{T}_\sigma^{\partial S,\partial S}\circ\mathrm{J}_z = -i\omega\mu_0\oint\limits_{\partial S} G_\sigma(\boldsymbol{\rho},\boldsymbol{\rho}')\mathrm{J}_z(\boldsymbol{\rho}')d\boldsymbol{\rho}',\ \boldsymbol{\rho}\in\partial S, \quad (3)$$

$$\mathcal{T}_\sigma^{S,\partial S}\circ\mathrm{J}_z = -i\omega\mu_0\oint\limits_{\partial S} G_\sigma(\boldsymbol{\rho},\boldsymbol{\rho}')\mathrm{J}_z(\boldsymbol{\rho}')d\boldsymbol{\rho}',\ \boldsymbol{\rho}\in S, \quad (4)$$

$$\mathcal{T}_\epsilon^{\partial S,S}\circ j_z = i\omega\iint\limits_{S} G_\epsilon(\boldsymbol{\rho},\boldsymbol{\rho}')j_z(\boldsymbol{\rho}')ds',\ \boldsymbol{\rho}\in\partial S. \quad (5)$$

III. Layered Media Green's Function Definition

Spatial domain expression for the Green's function $G_\epsilon(\boldsymbol{\rho},\boldsymbol{\rho}') = G_\epsilon(|x-x'|,y,y')$ can be written in the form of the following inverse Fourier transform over coordinate x [15]

$$G_\epsilon(|x-x'|,y,y') = \frac{1}{2\pi}\int\limits_{-\infty}^{+\infty}\hat{G}_\epsilon(k_x,y,y')e^{ik_x|x-x'|}dk_x, \quad (6)$$

where k_x is the spectral variable. The spectrum of the Green's function $\hat{G}_\epsilon(k_x,y,y')$ can be obtained in pole-residual form for arbitrary layered medium using technique described in [10] and [9]. The pole-residual definition of the spectrum allows to analytically evaluate the Fourier transform integral (6) producing the following space domain Green's function values at the discrete samples along the direction of stratification y for observation point elevations y_n and source point elevations y'_m

$$G_\epsilon(|x-x'|,y_n,y'_m) = \sum_{j=0}^{\mathcal{N}-1} T_{nj}R_{jm}d_m\frac{e^{-\sqrt{S_j}|x-x'|}}{2\sqrt{S_j}}, \quad (7)$$

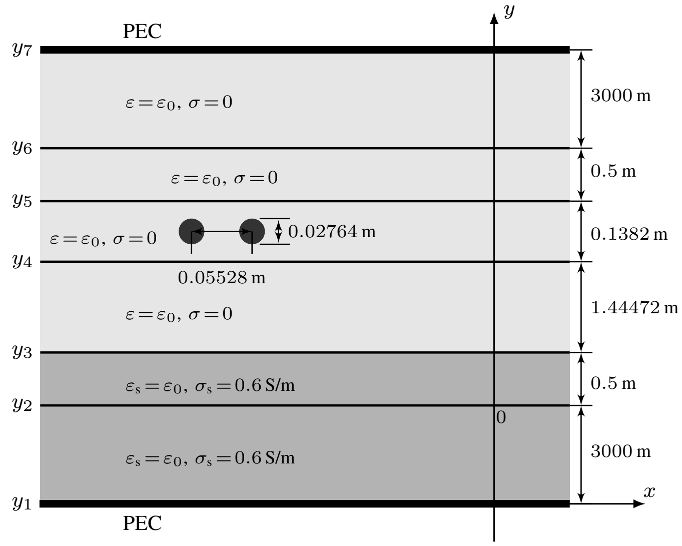

Fig. 1. Depiction of the two-conductor MTL with round conductors situated in two-layer medium consisting of air half-space and lossy half-space and its partitioning into layers. The layered medium is terminated by PEC planes situated at $y = \pm 3000$m.

In (7), the number of terms $\mathcal{N}$ in (7) is equal to the number of samples in the grid discretizing the cross-section of the layered medium when $|x-x'| = 0$ and drops rapidly with the increase of distance $|x-x'|$ between the source and observation points. Eigenvectors T_{nj} and R_{jm} and eigenvalues S_j are determined numerically [9].

IV. Numerical Results

In all the numerical studies two-conductor transmission line shown in Fig. 1 is considered. Each of the two round conductors has the diameter of 2.764cm and conductivity of copper $\sigma = 5.8\cdot 10^7$S/m. Both conductors are buried to the same depth in the soil or elevated to the same height above the air-soil interface (Fig. 1). The depth of burial or height of elevation vary in different experiments to investigate the impact of the soil on the impedance. The horizontal separation between the centers of the conductors is taken to be 0.05528m in all the experiments. Differential excitation ($V_1 = 1V$, $V_2 = 0V$) is applied to the conductors so that the (2×2) admittance matrix $\mathbf{Y}$ is obtained first by sampling the currents under impressed voltage excitation and subsequently inverted to obtain the impedance matrix as $\mathbf{Z} = \mathbf{Y}^{-1}$.

In order to produce pole-residual approximation for the layered medium Green's function spectrum $\hat{G}_\epsilon$ two-layer medium formed by the layer of air situated above half-space of soil with $\sigma_s = 0.6$S/m, $\varepsilon_s = \varepsilon_0$ and material interface located at $y = 0.5$m elevation (Fig. 1), the FD grid is introduced over the interval of interest $y \in [0, 0.5\text{m}]$ with the uniform step of 5.4945mm, with the uniform step of 15.876mm on the interval $y \in [0.5\,\text{m}, 1.94472\text{m}]$, and 5.4945mm on the interval $y \in [2.08292\text{m}, 2.58292\text{m}]$. The interval $y \in [1.94472\text{m}, 2.08292\text{m}]$ contains the conductors so the sampling step in this interval is determined by the size of the mesh elements discretizing the conductors cross-section.

TABLE I

IMPEDANCE OF TWO-CONDUCTOR MTL AT DIFFERENT FREQUENCIES COMPUTED USING COMSOL FEM, SVS-EFIE AND PSCAD FOR DIFFERENT ELEVATIONS OF THE MTL ABOVE THE SOIL-AIR INTERFACE. NEGATIVE ELEVATIONS CORRESPOND TO DEPTH OF MTL BURIAL IN SOIL

Solver	$Z_{1,1}$	$Z_{2,1}$	$Z_{1,1}$	$Z_{2,1}$
	$60Hz$		$1000Hz$	
COMSOL (in air)	$3.428E-05+9.426E-04i$	$-1.522E-08+8.219E-04i$	$1.144E-04+0.015i$	$-1.245E-06+0.014i$
COMSOL (5.0691m)	$8.819E-05+6.991E-04i$	$5.39E-05+5.784E-04i$	$8.321E-04+9.907E-03i$	$7.165E-04+8.142E-03i$
SVS-EFIE (5.0691m)	$8.828E-05+6.983E-04i$	$5.4E-05+5.778E-04i$	$8.298E-04+9.893E-03i$	$7.145E-04+8.132E-03i$
PSCAD (5.0691m)	$8.679E-05+7.036E-04i$	$5.484E-05+5.815E-04i$	$8.417E-04+9.991E-03i$	$7.375E-04+8.154E-03i$
COMSOL (1.51382m)	$9.165E-05+6.951E-04i$	$5.736E-05+5.744E-04i$	$9.982E-04+9.662E-03i$	$8.826E-04+7.898E-03i$
SVS-EFIE (1.51382m)	$9.178E-05+6.948E-04i$	$5.749E-05+5.741E-04i$	$9.955E-04+9.658E-03i$	$8.8E-04+7.894E-03i$
PSCAD (1.51382m)	$8.981E-05+7.003E-04i$	$5.785E-05+5.783E-04i$	$10.034E-04+9.778E-03i$	$8.992E-04+7.941E-03i$
COMSOL (-3.0691m)	$9.658E-05+6.897E-04i$	$6.229E-05+5.69E-04i$	$1.251E-03+9.319E-03i$	$1.136E-03+7.555E-03i$
SVS-EFIE (-3.0691m)	$9.589E-05+6.894E-04i$	$6.159E-05+5.688E-04i$	$1.248E-03+9.312E-03i$	$1.133E-03+7.549E-03i$
PSCAD (-3.0691m)	$9.439E-05+6.896E-04i$	$6.244E-05+5.675E-04i$	$1.243E-03+9.359E-03i$	$1.138E-03+7.521E-03i$
	$5000Hz$		$10000Hz$	
COMSOL (in air)	$2.468E-04+0.077i$	$-4.227E-06+0.069i$	$3.475E-04+0.155i$	$-5.121E-06+0.138i$
COMSOL (5.0691m)	$2.954E-03+0.046i$	$2.703E-03+0.037$	$4.913E-03+0.09i$	$4.56E-03+0.073i$
SVS-EFIE (5.0691m)	$2.944E-03+0.046i$	$2.695E-03+0.037$	$4.9E-03+0.09i$	$4.549E-03+0.073i$
PSCAD (5.0691m)	$2.984E-03+0.046i$	$2.762E-03+0.037$	$4.936E-03+0.09i$	$4.626E-03+0.072i$
COMSOL (1.51382m)	$4.184E-03+0.044i$	$3.933E-03+0.035i$	$7.634E-03+0.084i$	$7.281E-03+0.067i$
SVS-EFIE (1.51382m)	$4.17E-03+0.044i$	$3.921E-03+0.035i$	$7.621E-03+0.084i$	$7.269E-03+0.067i$
PSCAD (1.51382m)	$4.258E-03+0.0441i$	$4.037E-03+0.0352i$	$7.794E-03+0.084i$	$7.484E-03+0.067i$
COMSOL (-3.0691m)	$6.136E-03+0.04i$	$5.884E-03+0.032i$	$0.012+0.075i$	$0.012+0.058i$
SVS-EFIE (-3.0691m)	$6.124E-03+0.04i$	$5.875E-03+0.032i$	$0.012+0.075i$	$0.012+0.058i$
PSCAD (-3.0691m)	$6.109E-03+0.04i$	$5.888E-03+0.032i$	$0.012+0.075i$	$0.012+0.058i$

TABLE II

IMPEDANCE OF TWO-CONDUCTOR MTL AT 100000HZ COMPUTED USING COMSOL FEM [14], SVS-EFIE AND PSCAD [8] FOR DIFFERENT ELEVATIONS OF THE MTL ABOVE THE SOIL-AIR INTERFACE. NEGATIVE ELEVATIONS CORRESPOND TO DEPTH OF MTL BURIAL IN SOIL

Solver	$Z_{1,1}$	$Z_{2,1}$
	$100000Hz$	
COMSOL (in air)	$1.086E-03+1.117i$	$-1.708E-05+0.95i$
COMSOL (5.0691m)	$0.022+0.847i$	$0.021+0.681i$
SVS-EFIE (5.0691m)	$0.022+0.847i$	$0.021+0.681i$
PSCAD (5.0691m)	$0.0216+0.855i$	$0.0206+0.679i$
COMSOL (1.51382m)	$0.048+0.744i$	$0.047+0.578i$
SVS-EFIE (1.51382m)	$0.048+0.746i$	$0.047+0.579i$
PSCAD (1.51382m)	$0.0487+0.752i$	$0.0477+0.576i$
COMSOL (-3.0691m)	$0.099+0.59i$	$0.098+0.424i$
SVS-EFIE (-3.0691m)	$0.1+0.591i$	$0.099+0.424i$
PSCAD (-3.0691m)	$0.0992+0.596i$	$0.0981+0.421i$

The upper and lower buffer regions $y \in (2.58292\text{m}, 3000\text{m})$ and $y \in (-3000\text{m}, 0)$ are mapped to the parametric variable domain $t \in (0, 0.5)$ via transformations [9], respectively.

In order to obtain the reference solution with COMSOL's FEM, a 2-D model was created by selecting AC/DC Module and Magnetic and Electric Fields (mef) interface. In our simulation, we enclosed the entire model geometry in a circular perfect electrically conducting shield of 3000m radius (Fig. 4). The interface between air domain and the soil domain divides the circular region of 3000m redius into two sub regions each imitating half-space of the air and the soil. The resultant geometry mimics that depicted in Fig. 1. With this configuration, the air and soil domains are extended far enough so that the influence of the parallel perfect electrically conducting plates in SVS-EFIE solution becomes similar to the influence of the perfect electrically conducting shield terminating FEM mesh in COMSOL. Various numerical experiments with the circular PEC boundary radius in COMSOL ranging from 50m to 3000m have demonstrated that smaller radiuses of the mesh truncating boundary in FEM may cause notable error between the planar layered medium based SVS-EFIE results and FEM results obtained inside the circular PEC enclosure. This is especially the case at low frequencies when field extends many kilometers away from the transmission line. Upon the PEC boundary in FEM is taken sufficiently far, however, the numerical results from FEM can be used to verify the SVS-EFIE solution based on planar layered medium analysis. More details on setting up the COMSOL simulation can be found in [9]. From Fig. 2 and Fig. 3, we observe that the currents in the conductors computed using COMSOL's FEM are in good agreement with the currents obtained using proposed SVS-EFIE layered medium formulation at 60Hz and 5000Hz.

The impedance matrix at 60Hz, 1000Hz, 5000Hz, 10000Hz using layered medium formulation of the SVS-EFIE, COMSOL, and PSCAD are shown in Table I and for 100000Hz in Table II for various heights of elevation of the transmission line center above the soil/air interface. The negative levels of elevation correspond to the depth of burial of the transmission line in the soil. The impedance matrix values show a close agreement with the MoM solution of the layered medium SVS-EFIE. One can observe in Tables I and II the impact on the impedance matrix elements from the transmission line elevation above soil/air interface. The impact is clearly increasing with frequency. The currents in the conductors computed using SVS-EFIE and COMSOL at 60Hz and 5000Hz are shown in Fig. 2 and Fig. 3, respectively. Close agreement between the SVS-EFIE and FEM solutions is observed. The impedance values obtained with PSCAD [8] are also shown in the Tables I and II.

The distribution of the electric current density computed using COMSOL FEM solver in the air and the soil domains at 60Hz and 1000Hz are shown in Fig. 4 and Fig. 5, respectively, for the transmission line elevated $1.51382m$ above the air-soil interface. One can observe that at low frequencies the current induced in the soils extends several kilometers away from the transmission line. As the frequency increases, the current becomes more localized to the region near the transmission line.

It's important to point out that in the frequency range of conducted experiment (60Hz-100,000Hz) the relative permittivity ϵ_r of the soil has negligible impact compared to its conductivity. The reason for that is the negligible displacement current $|j\omega\epsilon_0\epsilon_r\mathbf{E}|$ in the soil compared to the current of conductivity $|\sigma\mathbf{E}|$. This due to the fact that $\omega\epsilon_0\epsilon_r$ remains to be much smaller than σ in the frequency range of interest. Namely, the highest possible value of $\omega\epsilon_0\epsilon_r$ term is attained at frequency of 100kHz in our experiments. Assuming $\epsilon_r = 10$, we have $\omega\epsilon_0\epsilon_r = 2\pi \cdot 100,000 \cdot 8.85 \cdot 10^{-12} \cdot 10 = 5.56 \cdot 10^{-5}$. Conductivity of soil, however, is taken at 0.6S/m. One can see that $\omega\epsilon_0\epsilon_r$ remains over 10000 times smaller than σ in all of our experiments. Indeed, if the frequencies reach 1MHz and and/or dry soils with $\sigma < 0.01$S/m are considered, the impact from ϵ_r may become substantial and have to be accounted for as discussed in [16].

V. Conclusions

This paper describes a novel rigorous method for broadband analysis of power delivery transmission lines in the presence of the soil/air half-space and general layered medium. The approach is based on Method of Moments solution of the multilayered medium formulation of the Surface-Volume-Surface Electric Field Integral Equation (SVS-EFIE). To demonstrate the method and its accuracy in the broad range of frequencies investigation of the impact from the half-space formed by the air-soil interface on the impedance matrix of the two-conductor overhead and buried transmission lines is conducted. Finite Element Method (COMSOL) computations of the same impedance matrix elements is performed for validation of the proposed SVS-EFIE solution. Numerical experiments are conducted at both low and high frequencies in the range from 60Hz to 100kHz. It is demonstrated that the impact of the soil presence is significant for both buried cables and overhead lines and grows with frequency. In all the considered scenarios SVS-EFIE solution is shown to provide closely matching results with COMSOL FEM solutions.

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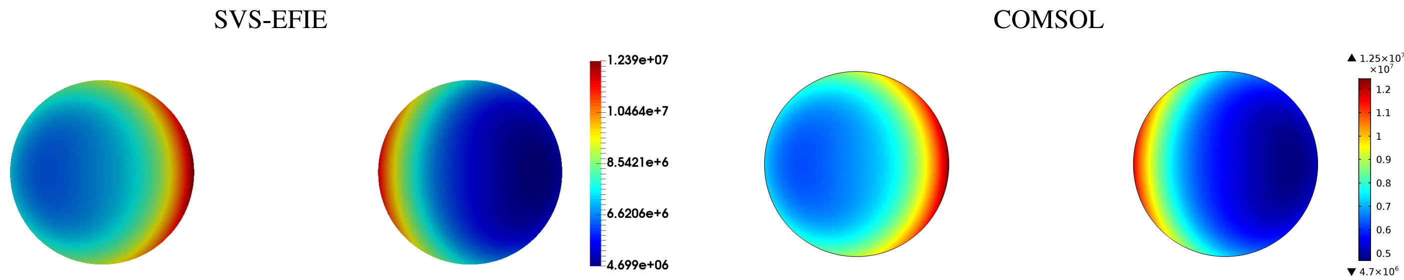

Fig. 2. Volumetric current density in the two-conductor MTL at 60Hz. The centers of conductors are elevated 1.51382m above the soil/air interface. The conductors are driven differentially 1V in the left conductor and 0V in the right conductor. The current distribution computed using SVS-EFIE is shown in the left. The validated result by COMSOL FEM [14] is shown in the right.

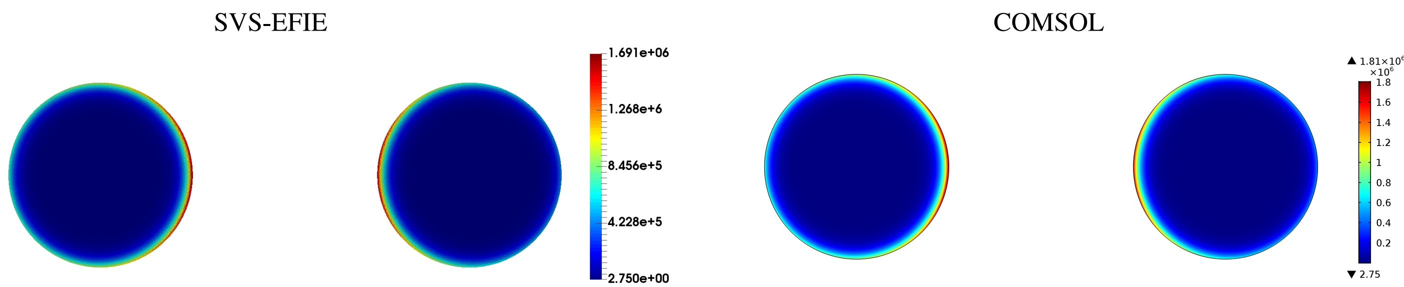

Fig. 3. Volumetric current density in the two-conductor MTL at 5000Hz. The centers of conductors are elevated 1.51382m above the soil/air interface. The conductors are driven differentially 1V in the left conductor and 0V in the right conductor. The current distribution computed using SVS-EFIE is shown in the left. The validated result by COMSOL FEM [14] is shown in the right.

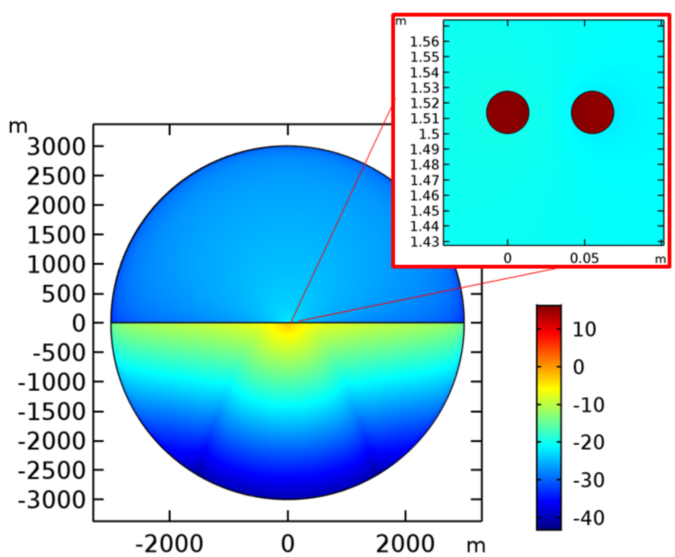

Fig. 4. Model set up and the current density distribution in the conductors and the soil domain computed in COMSOL FEM [14] at 60Hz.

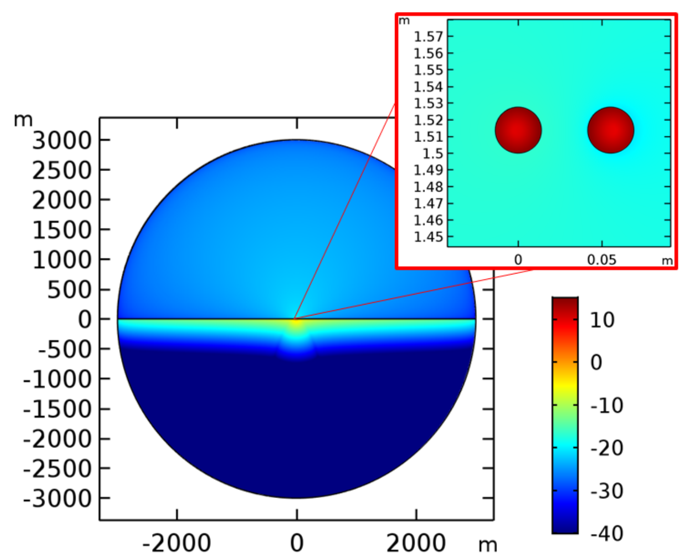

Fig. 5. Model set up and the current density distribution in the conductors and the soil domain computed in COMSOL FEM [14] at 1000Hz.