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Sensitivity Analysis of Frequency-Dependent Soil Models based on Rational Approximation

J. P. L. Salvador, A. C. S. Lima, R. Alipio, M. T. Correia de Barros

Abstract—**The rational approximation of frequency-dependent soil models is a straightforward way to deal with ground parameters variation, once the sum of partial fractions in frequency-domain become a sum of exponentials in time-domain. This work proposes to investigate sensitivity analyses of the frequency-dependent soil model proposed by Alipio and Visacro with respect to frequency and to the dc conductivity, aiming to a rational realization that is actually simpler than the one obtained through a traditional fitting technique, namely, Vector Fitting (VF). Results indicated that a simpler realization is possible using a smaller subset of the poles and residues obtained via VF, without compromising accuracy within the frequency range of interest.**

Keywords—**soil parameters; soil modeling; sensitivity analysis; grounding.**

I. Introduction

An accurate assessment of the transient behavior of the ground during lightning related phenomena is of outmost importance for the design and operation of any power system. In the past, the ground was assumed as a good conductor and a little attention has been given to its season variability and the uncertainty regarding the actual ground measurements. However, complexity in topology of the power networks has increased considerably thus requiring improved modeling for a precise evaluation of the actual stage of any given network during and after surge occurrences. Therefore, there is a solid demand for better, more accurate soil models.

The frequency dependence of soil parameters, in particular the soil conductivity and permittivity, have been a subject of intense research for quite some time now [1]–[9]. An overview of the characteristics of most of the soil models is presented in [10]. Shortly after this reference, Alipio and Visacro have proposed in [11] a causal soil model that could be generalized depending on the “nature” of the response, i.e., conservative, relatively conservative or mean.

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An issue for the inclusion of frequency-dependent soil models is their time-domain implementation, especially in studies directly based on the solution of Maxwell equations, since most soil models do not have a closed-form inverse transform. Even in the case where a closed-form transform can be obtained, as in [12], the resultant convolution operations are computationally costly. One could thus resort to Finite Differences in Time-Domain (FDTD) [13] or consider a rational approximation in the frequency domain as it leads to closed-form expression in the time-domain in a straighforward way, by means of a sum of exponentials that can be solved in time domain with recursive convolutions [12].

In a previous paper a comparison between rational realizations of the Alipio-Visacro (AV) and Smith-Longmire (SL) soil models was presented. The authors have identified that the rational approximation can lead to rather compact realization as there is a well-defined ratio between poles and residues [14]. In this work, we aim to further investigate this characteristic and to identify the dominance of the poles considered for the rational approximation. To do so, a sensitivity analysis is carried out in frequency-domain to identify which set of rational function is more dominant, depending on the frequency region of interest and time-domain results are presented for accuracy assessment. Since the AV model showed to be more general, allowing its rational realization with a unique universal set of poles regardless the soil resistivity, it is assumed as reference in this paper.

This paper is organized as follows: Section II summarizes the characteristics of a frequency-dependent soil including the rational approximation considering the Alipio-Visacro (AV) soil model. The sensitivity analysis is carried out in Section III. Based on the sensitivity results a minimal realization is proposed in Section IV. Section V presents the time-domain results and the main conclusions of this work are detailed in Section VI. Appendix A summarizes the characteristics of the AV soil Model.

II. Soil Modeling

A. Frequency-Dependent Soil Parameters

The frequency dependence of soil parameters can be defined from the relation between the electric field $\mathbf{E}$ and magnetic field $\mathbf{H}$, which has the following behavior in the frequency-domain:

$$\nabla \times \mathbf{H} = \left[\sigma'(\omega) + \omega\varepsilon''(\omega) + j\omega\left(\varepsilon'(\omega) - \frac{\sigma''(\omega)}{\omega}\right)\right]\mathbf{E} \quad (1)$$

with

$$\sigma'(\omega)+\omega\varepsilon''(\omega)+j\omega\left(\varepsilon'(\omega)-\frac{\sigma''(\omega)}{\omega}\right)=\sigma_{eff}(\omega)+j\omega\varepsilon_{eff}(\omega)$$

and where σ', σ'', ε', ε'' are all real-valued functions of the angular frequency ω, and σ_{eff} and ε_{eff} are, respectively, the so-called effective conductivity and permittivity. The parameter $\sigma''(\omega)$ is ultimately related to conduction currents at phase quadrature to the electric field. Although such currents can be found in some materials at optical frequencies [15], considering the frequency range of interest in the calculation of transients in electrical systems, conduction currents in the ground can be assumed to be in phase with the electric field and, therefore, $\sigma''=0$. Furthermore, considering experimental results of frequency dependence of the electrical parameters of real soils [11], [16], $\sigma'=\sigma_0$, where σ_0 is a real number corresponding to the soil dc conductivity and physically express its ability to transportac electric charges when an electric field is applied, and naturally is responsible for the losses associated with this process.

The real parcel of permittivity (ε') expresses the material ability to be polarized and to store electric energy, when an electric field is applied. The imaginary parcel (ε'') is associated with the losses occurring during the polarization process. Such losses represent the part of energy of the applied field, which is dissipated as heat because of the friction experienced by the electric dipoles as they continuously move in response to the alternating field. These two parcels are not independent and share a causal relationship given by the so-called Kramers-Krönig's relation given by

$$\varepsilon'(\omega)=\varepsilon_\infty+\frac{2}{\pi}\int_0^\infty\frac{\omega'\varepsilon''(\omega')}{\omega'^2-\omega^2}d\omega' \tag{2}$$

$$\varepsilon''(\omega)=-\frac{2\omega}{\pi}\int_0^\infty\frac{\varepsilon'(\omega')-\varepsilon_\infty}{\omega'^2-\omega^2}d\omega' \tag{3}$$

where ε_∞ is a real constant parameter related to the asymptotic behavior of the permittivity at higher frequencies, ω' is the integration variable (real).

Considering the complex permittivity and the Kramers-Krönig's relations, it is possible to rewrite (1) as

$$\nabla\times\mathbf{H}=\kappa(s)\mathbf{E} \tag{4}$$

where

$$\kappa(s)=\sigma_0+s\left[\varepsilon_\infty+\overline{\kappa}(s)\right] \tag{5}$$

The frequency dependence in both $\varepsilon'(\omega)$ and $\varepsilon''(\omega)$ is included in $\overline{\kappa}(s)$, where $s=j\omega$. As shown in [12] and later in [14], a rational approximation of $\overline{\kappa}$ in the frequency-domain is feasible and accurate as it will be detailed next.

B. Rational Modeling

In this work, initially we have used the so-called Vector Fitting (VF) [17]–[20] routine for the fitting which implies in rewriting (4) as (5) and

$$\overline{\kappa}(s)=\sum_{i=1}^{N}\frac{K_i}{s-p_i} \tag{6}$$

A question that this approach brings is the number of poles/residues, i.e., the order of approximation to be considered. In a previous work [14], we have obtained a rms error several orders of magnitude below the actual function, regardless of the value of the dc soil conductivity and the permittivity at the very high frequencies, i.e., σ_0 and ε_∞. It should be pointed out that only real poles and residues were obtained in the fitting process. In this procedure it is as if the soil is to be represented by a ladder of RC circuits. It was also found that the relation between the residues and poles remained constant regardless of the soil dc resistivity considered. Table I presents the relation between the dc soil resistivity, i.e. $\rho_0=(1/\sigma_0)$, and the obtained first pole and first residue.

For this soil model it is verified that the same poles are found regardless the value of the dc soil resistivity [14]. Table II shows the relation between residue K_i (for $i\geq 2$) and the first residue. This table also presents the relation between pole p_i (also for $i\geq 2$) and the first pole. Thus, the rational approximation allows for a rather concise formulation of frequency dependent soil parameters as we need only to define the value of the residue associated with the dc conductivity and the relation presented in Table II can be used to define the other residues.

TABLE I
FIRST POLE AND RESIDUE AS A FUNCTION OF THE SOIL DC RESISTIVITY.

ρ_0 [Ω.m]	K_1	p_1
100	7.45×10^0	-2.24×10^{13}
500	4.83×10^0	-2.24×10^{13}
1000	4.00×10^0	-2.24×10^{13}
3000	2.98×10^0	-2.24×10^{13}
5000	2.59×10^0	-2.24×10^{13}

It can be noticed that the function has poles in a decreasing order and the same can be said about the residues. Thus, it might be possible to explore this feature to derive a lower order realization suitable for some particular analysis. For instance, if one is interested in switching surges where the frequency band is well-defined, a lower order might suffice for this analysis. The results obtained in Table I along with further fittings, i.e., calculating new sets of residues for new dc conductivities, lead to (6), which describes a straightforward relation between K_1 and σ_0:

$$K_1=4.002\ \sigma_0^{0.2699} \tag{7}$$

Thus, together with the ratio presented in Table II, a rather general frequency-dependent soil model can be derived using rational approximation. It must be emphasized that the rational approximation demands only the knowledge of the low frequency soil conductance, i.e. σ_0, for its realization.

TABLE II
RELATION BETWEEN RESIDUES AND POLES FOR THE RATIONAL APPROXIMATION.

#	K_i/K_1	p_i/p_1
2	2.56×10^{-1}	2.62×10^{-1}
3	1.32×10^{-1}	1.05×10^{-1}
4	8.26×10^{-2}	4.45×10^{-2}
5	5.33×10^{-2}	1.87×10^{-2}
6	3.42×10^{-2}	7.63×10^{-3}
7	2.16×10^{-2}	2.99×10^{-3}
8	1.33×10^{-2}	1.12×10^{-3}
9	8.03×10^{-3}	4.02×10^{-4}
10	4.72×10^{-3}	1.36×10^{-4}
11	2.69×10^{-3}	4.32×10^{-5}
12	1.49×10^{-3}	1.28×10^{-5}
13	7.89×10^{-4}	3.50×10^{-6}
14	4.01×10^{-4}	8.66×10^{-7}
15	1.93×10^{-4}	1.90×10^{-7}
16	8.72×10^{-5}	3.58×10^{-8}
17	3.62×10^{-5}	5.47×10^{-9}
18	1.33×10^{-5}	6.02×10^{-10}
19	4.05×10^{-6}	3.53×10^{-11}

Figure 1 illustrates the behavior of the rational approximation together with the original data using the AV soil model for two values of the dc soil resistivity presented in Table I. The mismatches between the rational approximation and the original data are roughly 3 orders of magnitude below the original data as it can be observed from the figure. Although not shown here, the results for the other soil dc resistivities presented in Table I had a similar performance.

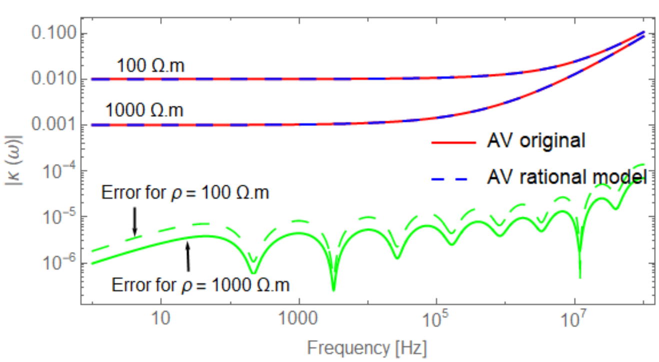


Fig. 1. Rational approximation of the AV soil model.

III. SENSITIVITY ANALYSIS

In this section we investigate the sensitivity of the rational approximation with respect to two parameters, namely, with respect to the low frequency soil conductivity, σ_0, and with respect to the complex frequency, s.

A. w.r.t the low frequency conductivity σ_0

As mentioned earlier, the value of σ_0 plays a fundamental role in the definition of the rational approximation. Thus, it is valuable to determine its impact in the realization. Here, we use the simple definition of sensitivity as shown below, i.e.,

$$S(\sigma) = \frac{\partial \kappa(s)}{\partial \sigma} = \left(1 + \sum_{i=1}^{N} \frac{\frac{\partial K_i}{\partial \sigma}}{s - p_i} \right) \tag{8}$$

where

$$\frac{\partial K_i}{\partial \sigma} = 0.167409 \left(\frac{1}{\sigma_0} \right)^{0.7301} \frac{K_i}{K_1} \tag{9}$$

One drawback in using directly the value of the sensitivities is the possible widespread value that a derivative might assume in the region of interest. Thus, from a practical point of view it is more convenient to use the relative sensitivity as in [21]. The relative sensitivity in this case can be defined as

$$S_{rel}(\sigma) = \frac{1}{|\kappa(s)|} \left| \frac{\partial \kappa(s)}{\partial \sigma} \right| \tag{10}$$

The results for this relative sensitivity are shown in Fig. 2 considering distinct values for σ_0, as shown below in Table III. As expected, the soil with the highest conductivity presented the lowest sensitivity. Furthermore, it can be observed that in all configurations regardless of the value of σ_0 a rather constant relative sensitivity is observed up to a well-defined frequency. The lower the value of σ_0, lower is the maximum frequency where $S_{rel}(\sigma)$ can be assumed constant.

TABLE III
VALUES FOR σ_0 IN [S/M].

σ_0^1	σ_0^2	σ_0^3	σ_0^4
0.01	0.002	0.001	0.00025

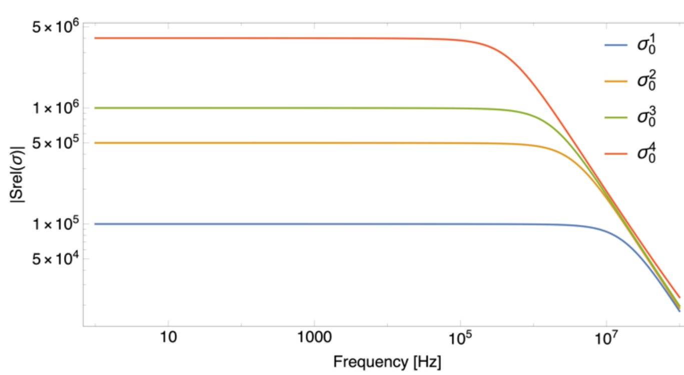


Fig. 2. Sensitivity w.r.t the low frequency soil conductivity considering distinct values for σ_0.

B. w.r.t the complex frequency s

For the frequency-domain analysis, we consider the sensitivity as the variation of the soil immittance with the complex frequency s, i.e.,

$$S(s) = \frac{\partial \kappa(s)}{\partial s} = \left[e + \sum_{i=1}^{N} \frac{K_i \cdot p_i}{s - p_i} - \sum_{i=1}^{N} \frac{K_i \cdot s}{(s - p_i)^2} \right] \tag{11}$$

The relative sensitivity is also evaluated with respect to the complex frequency s and defined by (12). The results for the relative sensitivity are depicted in Fig. 3.

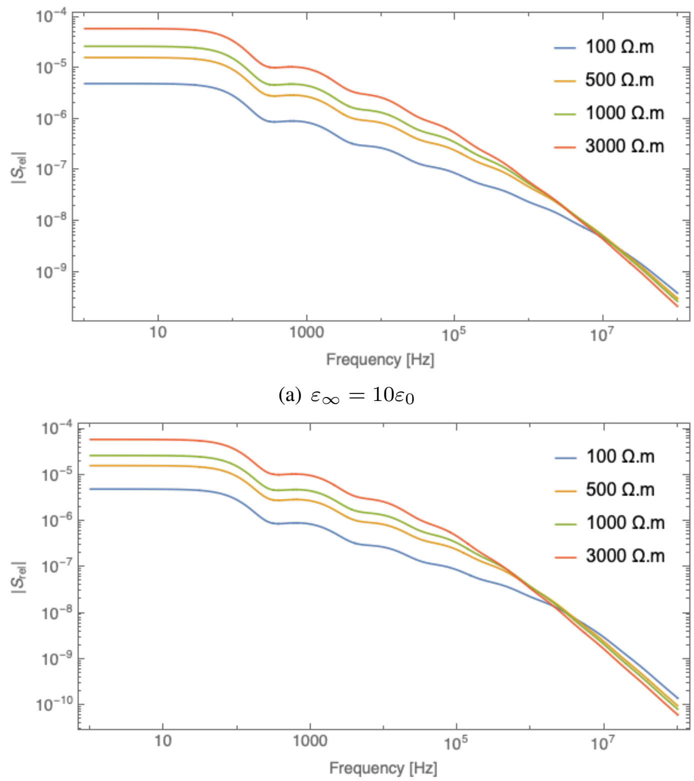
$$S_{rel}(\sigma) = \frac{1}{|\kappa(s)|}\left|\frac{\partial\kappa(s)}{\partial s}\right| \tag{12}$$

It is noticeable now that soils with higher dc resistivity are more affected by the inclusion of the frequency-dependent parameters. For higher frequencies, all sensitivities tend to similar values. This can be understood as in this region the contribution of the rational terms is neglectable, thus all the models behave as if

$$\kappa(s) \approx \sigma_0 + s\varepsilon_\infty \tag{13}$$

These results also indicated that the value of ε_∞ has an important effect as it is defined at the higher frequency range. For instance, for $\varepsilon_\infty = 10\varepsilon_0$ is roughly defined by 10 MHz while if $\varepsilon_\infty = 40\varepsilon_0$, this region starts at 2 MHz. It could be postulated that the highest frequency thus approximately the inverse ratio of the values used for high frequency permittivity. It should be pointed out that this analysis is an extrapolation of the AV model as it is defined for frequencies up to 4 MHz. Nevertheless, these results indicate that for higher frequencies the inclusion of the soil parameters frequency dependence might no longer be required.

It is worth mentioning that the frequency threshold for the definition of the high frequency range depends approximately by the inverse of the relation between the permittivities. Furthermore, the behavior depicted shows a strong tendency caused by one of the poles, i.e., once the first attenuation occur the curve does not oscillate very often and follows almost the same pattern up to higher frequencies.



(a) $\varepsilon_\infty = 10\varepsilon_0$

(b) $\varepsilon_\infty = 40\varepsilon_0$

Fig. 3. Relative Sensitivity of the rational approximation of the AV soil model considering distinct values of σ_0 and ε_∞.

IV. Minimum Realization

The usage of the so-called dominant poles [22] has been used for the pole-residue realization of linear power systems based on analytical expression of the admittance in the frequency domain. It is based on a Newton-Raphson scheme and can be used to obtain pole and residue from an analytical and continuous expressions. Although this procedure cannot be directly applied to the AV soil model, some of the fundamentals can be used as parametric tool in order to investigate different aspects of its formulation.

When sensitivity to the complex frequency s was performed, it was seen that, for a certain range of frequencies, the term of the rational realization corresponding to the sum of partial fractions tends to diminish its importance. One of the dominant pole fundamentals is to investigate the residue over pole ratio, i.e.,

$$\lim_{s\to 0}\frac{K_i}{s-p_i} \tag{14}$$

One interesting aspect of this approach is that it can be used to evaluate the numerical performance of simpler expressions, i.e., considering a lower number of residues and poles in the rational approximation. Although the residues are strictly related to the dc conductivity whereas the poles remain the same, we are interested in the order of magnitude of the ratio residue/pole.

As it was observed in Fig. 3, the behavior of the curves is very close to a first order system. The main difference lies in the upper frequency limit where the relative sensitivity no longer is constant. An investigation in the performance of the rational approximation indicates an interesting feature. Fig. 4 shows the ratio between the complete rational model and a minimal rational realization considering only the first residue and pole, i.e., those with larger magnitude. It is calculated as

$$\Delta = \left|\frac{\kappa(s)}{\sigma_0 + s\left(\varepsilon_\infty + \dfrac{K_1}{s+p_1}\right)}\right| \tag{15}$$

As it can be seen, the minimal realization is fairly accurate for frequencies below 1 kHz, although it gives good results for little higher frequencies assuming low-conductive soils. In the frequency range between tens of kHz to few MHz the errors increase, especially in the case if high-resistivity soils. Finally, as the frequency increases above 10 MHz the errors tend to reduce, since the immittance behavior is approximately dominated by $\sigma_0 + s\varepsilon_\infty$.

Further investigation is necessary to identify the realization that provides results accurate enough through a larger frequency range than the minimal realization. As an illustration, see Table IV, with ratio residue/pole calculated considering a dc resistivity ($1/\sigma_0$) of 1,500 Ω.m for the soil,

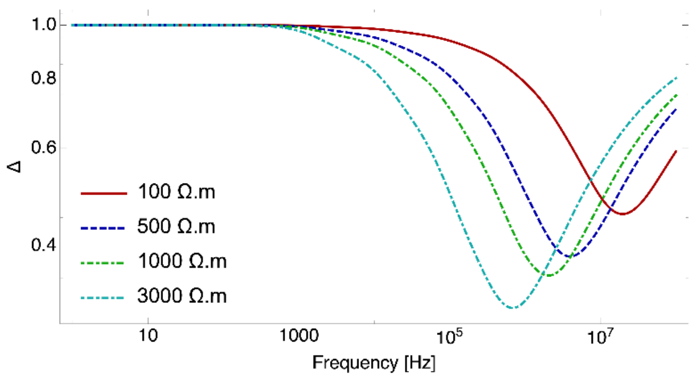


Fig. 4. Ratio between AV complete model and considering 1-pole realization.

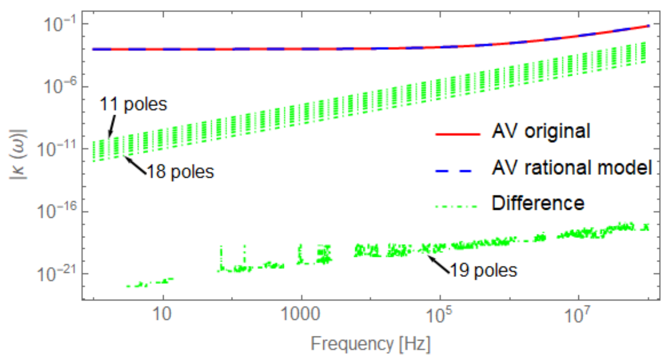


Fig. 5. Rational approximation considering simpler expressions.

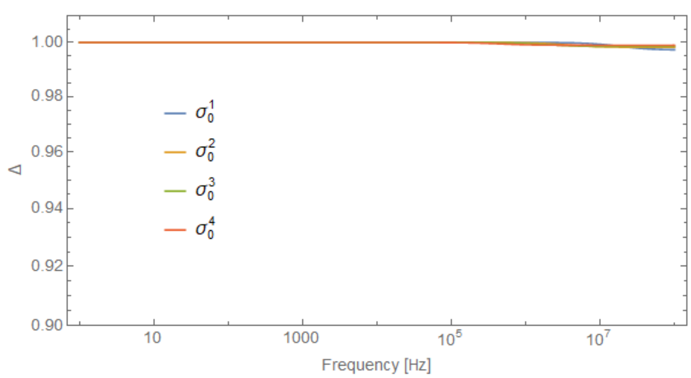


Fig. 6. Ratio between AV complete model and considering 11-pole realization.

where the larger poles impose smaller ratio. Similar results are to be found regarding soils with dc resistivity up to 15,000 Ω.m.

The difference between these ratios reaches 5 orders of magnitude between the first and the 19th pole. However, several tests were carried out considering the realization with as less poles as possible and, as it is noted in Fig. 5, the realizations with 11 to 18 poles present interesting results, i.e., produce fitted functions with small rms error. Therefore, even though the rational model is more precise with 19 poles, the realization with 11 of these poles seems to be suited enough.

Furthermore, the plot of Δ now considering an 11-pole realization is presented in Fig. 6, i.e. the evaluation is similar to (15) only considering an 11-term sum in the denominator. Unlike Fig. 4, when only one pole was used, this realization is fairly accurate throughout the frequency range of interest.

TABLE IV
RATIO RESIDUES/POLES CONSIDERING DC RESISTIVITY OF 1500 Ω.M.

K_1/p_1	-1.8365×10^{-8}
K_2/p_2	-3.5366×10^{-9}
K_3/p_3	-1.0586×10^{-9}
K_4/p_4	-3.8968×10^{-10}
K_5/p_5	-1.6265×10^{-10}
K_6/p_6	-7.4204×10^{-11}
K_7/p_7	-3.6146×10^{-11}
K_8/p_8	-1.8580×10^{-11}
K_9/p_9	-9.9748×10^{-12}
K_{10}/p_{10}	-5.5605×10^{-12}
K_{11}/p_{11}	-3.2014×10^{-12}
K_{12}/p_{12}	-1.8977×10^{-12}
K_{13}/p_{13}	-1.1533×10^{-12}
K_{14}/p_{14}	-7.1701×10^{-13}
K_{15}/p_{15}	-4.5594×10^{-13}
K_{16}/p_{16}	-2.9727×10^{-13}
K_{17}/p_{17}	-2.0264×10^{-13}
K_{18}/p_{18}	-1.5665×10^{-13}
K_{19}/p_{19}	-1.6021×10^{-13}

V. TIME-DOMAIN RESULTS

In order to assess the accuracy and illustrate the application of the proposed minimal rational realizations for frequency-dependent soil modeling, this section considers a well-known electromagnetic problem. It consists of a parallel-plate capacitor shown in Fig. 7 filled with a soil of dc resistivity ρ_0 and subjected to an impulse voltage $v(t)$ given by

$$v(t) = V_0(e^{-at} - e^{-bt}) \tag{16}$$

where V_0 is a constant related to the voltage amplitude and a and b are time constants related to the wave front and tail, respectively.

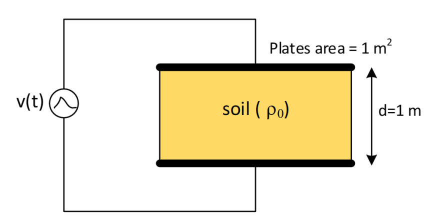
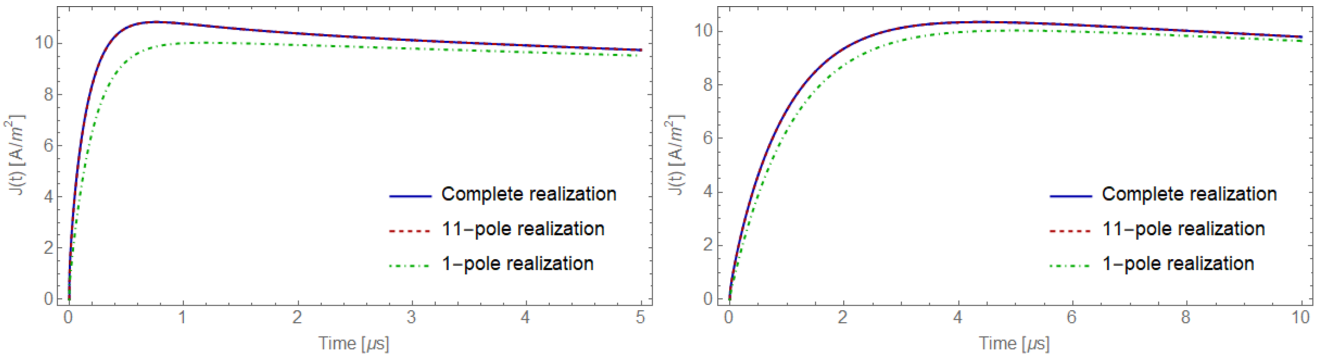


Fig. 7. Parallel-plate capacitor subjected to an impulse voltage $v(t)$.

The problem consists of determining the resulting transient current density $\mathbf{J}(t)$ between the capacitor plates. Such current density is given by the right side of the Ampere-Maxwell equation (1) which, considering the proposed rational approximation for the electrical parameters of soil, can be written in time-domain as

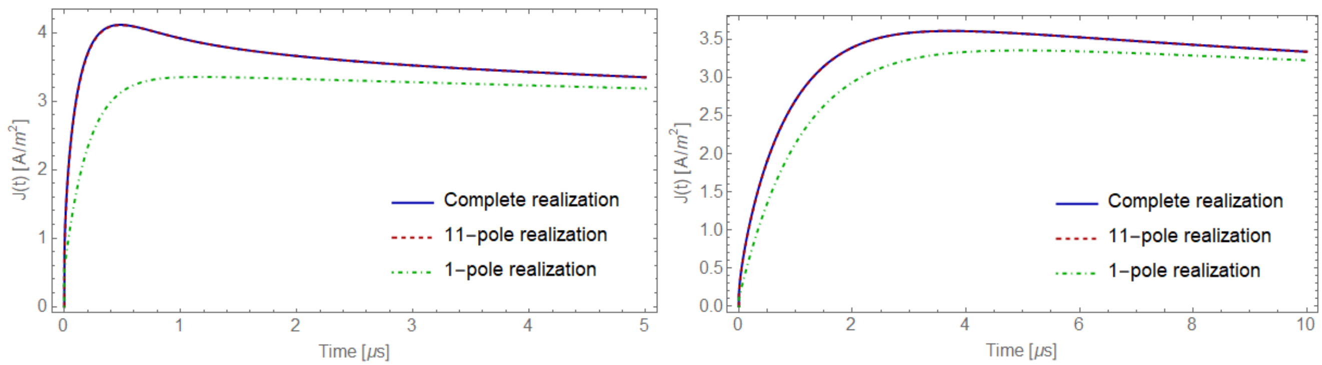
$$\mathbf{J}(t) = \sigma_0\mathbf{E}(t) + \varepsilon_\infty\frac{\partial\mathbf{E}}{\partial t} + \left(\sum_{i=1}^{N} K_i\ e^{p_i t}\right) * \frac{\partial\mathbf{E}}{\partial t} \tag{17}$$

where $\mathbf{E}(t)$ is the transient electric field between the plates which is assumed to be $\mathbf{E}(t) = v(t)/d$, neglecting the edge effects.



(a) Fast-fronted $v(t)$ (b) Slow-fronted $v(t)$

Fig. 8. Results considering $\rho_0 = 100\ \Omega$.m.



(a) Fast-fronted $v(t)$ (b) Slow-fronted $v(t)$

Fig. 9. Results considering $\rho_0 = 300\ \Omega$.m.

In the simulations, four different soils filling the parallel-plate capacitor were considered, namely, $\rho_0 = 1/\sigma_0 = 100$, 300, 1000 and 3000 Ω.m; two distinct 1 kV normalized impulse voltages were applied, namely, a fast-fronted one $1.2/50\ \mu$s and a slow-fronted one $5/75\ \mu$s, with parameters presented in Tab. V. All convolutions arising from (17) were recursively computed.

TABLE V
IMPULSE VOLTAGES PARAMETERS.

Parameter	Fast-Front	Slow-Front
V_0 [V]	1.0203×10^3	1.0633×10^3
a [s^{-1}]	1.4294×10^4	1.0061×10^4
b [s^{-1}]	4.8742×10^6	9.1119×10^5

Figs. 8, 9, 10 and 11 depict the obtained transient current densities between the parallel-plate capacitor filled with soils of 100, 300, 1000 and 3000 Ω.m, respectively, computed considering different rational realization for the electrical parameters of soil. According to the results, it is seen that regardless of the soil resistivity and the applied voltage signal, the 11-pole realization provides results basically identical to the full 19-pole realization proposed in [14], denoting the consistency of the proposed minimal rational realization.

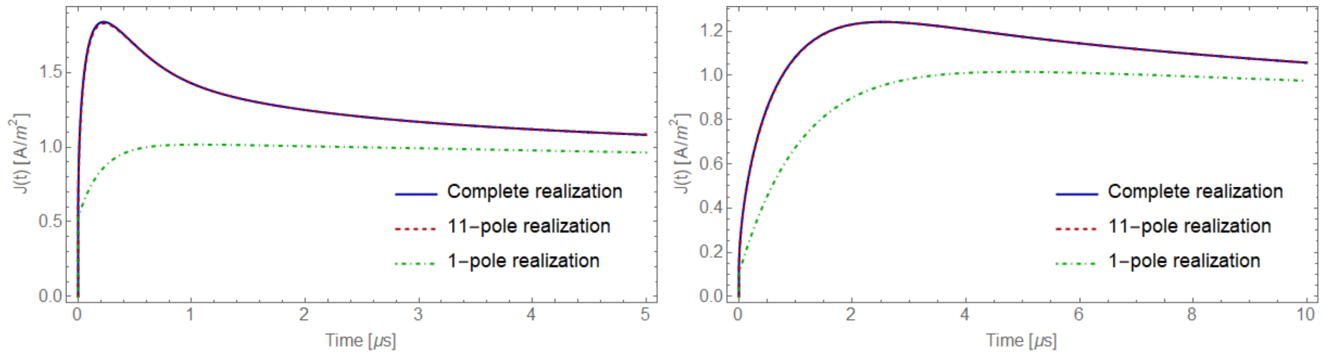
It is also seen that for low-resistivity soils, the very simple 1-pole realization gives fairly good results, especially considering the slow-fronted voltage wave. These results denote the feasibility of the proposed approach for rational realization of frequency-dependent electrical parameters of soil. Finally, depending on the soil resistivity, the use of the proposed minimal realizations can result in a huge reduction in computational cost, for instance, in the use of methods such as FDTD.

VI. Conclusion

This paper presented a discussion on sensitivity analyses of a rational approximation of frequency dependent soil model base on the expression proposed by the Alipio-Visacro soil model.

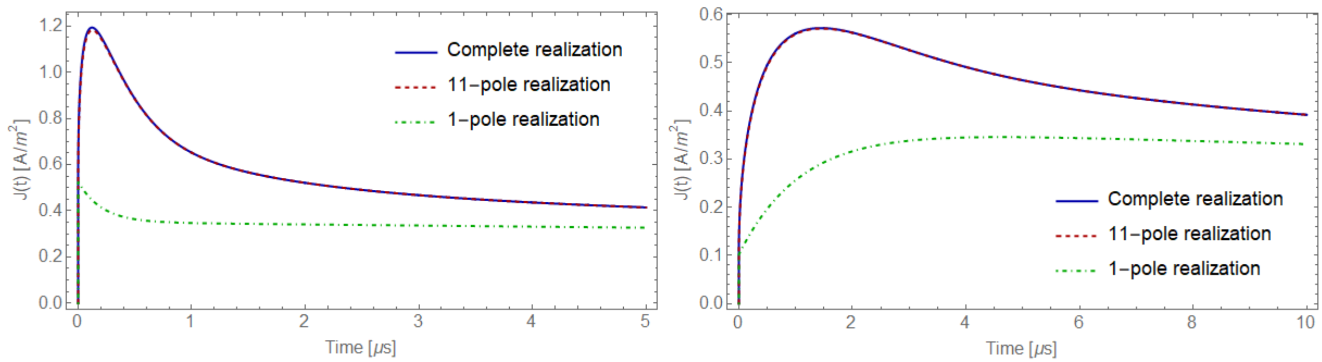
The sensitivity was carried out with respect to the low frequency soil conductivity and with respect to the complex frequency. The results indicate a clear dominance of some poles in the realization of the approximation. It was shown that for a limited bandwidth a very simple first order soil model can be used. Furthermore, it was shown that by relying of the sensitivity analysis it was possible to obtain a lower order realization without a significant loss of accuracy for the frequency range of interest in lightning protection.

A simple example of a time-domain transient calculation showing the accuracy of the proposed minimum realizations was presented. It was shown that, in agreement with the sensitivity analysis, the simple 1-pole representation of the model provided interesting results and, especially, both 11-pole and 19-pole realizations provided consistent results regardless of the soil dc resistivity considered.



(a) Fast-fronted $v(t)$ (b) Slow-fronted $v(t)$

Fig. 10. Results considering $\rho_0 = 1000$ Ω.m.



(a) Fast-fronted $v(t)$ (b) Slow-fronted $v(t)$

Fig. 11. Results considering $\rho_0 = 3000$ Ω.m.

Appendix – AV Soil Model

There are several experimentally obtained formulas for modeling the frequency dependence of soil parameters. In this paper, the Alipio-Visacro model is considered, which is based on the measurement of the frequency response of 65 type of soils, which presented low-frequency resistivity values ranging from 60 to about 18,000 Ω.m [11]. This model satisfies causality and was recently suggested in the CIGRE Brochure to take into account the frequency dependence of soil parameters in lightning related studies [23]. According to AV soil model, the effective soil conductivity, $\sigma_{eff}(f)(10^{-3}S/m)$, and permittivity, $\varepsilon_{eff}(f)$ [F/m], at a given frequency f [Hz] can be calculated using the following formulas [11]:

$$\sigma_{eff}(f) = \sigma_0 + \sigma_0 \times h(\sigma_0)\left(\frac{f}{10^6}\right)^{\xi} \tag{18}$$

$$\varepsilon_{eff}(f) = \varepsilon_\infty + \frac{\tan(\xi\pi/2)\times 10^{-3}}{2\pi\varepsilon_0 10^{6\xi}}\sigma_0 \times h(\sigma_0) f^{\xi-1} \tag{19}$$

where σ_0 is the DC conductivity. According to [11], the parameters ξ and ε_∞ along with the function $h(\sigma_0)$ can be chosen to account for the natural statistical dispersion of the frequency dependence of soil parameters. In this work, the following parameters are used in (18) and (19) to obtain mean results for the frequency variation of σ_{eff} and ε_{eff} [11]: $\xi = 0.54$, $\varepsilon_\infty = 12\varepsilon_0$ and $h(\sigma_0) = 1.26 \times \sigma_0^{-0.73}$.

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