Modal Propagation Characteristics and Transient Analysis of Multiconductor Cable Systems Buried in Lossy Dispersive Soils

T. A. Papadopoulos, Z. G. Datsios, A. I. Chrysochos, P. N. Mikropoulos, G. K. Papagiannis

Abstract--In surge analysis, an important issue is the influence of the imperfect earth on the propagation characteristics of the conductors. In this paper, the wave propagation characteristics and the transient performance of underground multiconductor cable systems in flat, vertical and trefoil arrangement are investigated. The Longmire and Smith frequency-dependent (FD) soil model as well as a generalized earth formulation are considered, taking into account in the analysis the impact of earth conduction effects on both the series impedance and shunt admittance of the cable conductors and sheaths. Comparisons are carried out with approximate earth formulations, neglecting the influence of imperfect earth on shunt admittances. Finally, resonance frequency analysis and transient simulations are performed for the different cable arrangements to evaluate the importance of FD soil modeling and earth formulation per cable arrangement.

Keywords: Earth conduction effects, electromagnetic transients, frequency-dependent soil properties, modal analysis, power cables, wave propagation.

I. INTRODUCTION

THE propagation characteristics and the transient performance of multiconductor cable systems can be analyzed in terms of natural modes of propagation; these are calculated on the basis of the per-unit length impedance and admittance matrices of the cable systems by applying proper modal transformations [1], [2]. The modal propagation characteristics of different cable configurations and their sensitivities to the electromagnetic (EM) and geometrical properties of the system under study have been systematically investigated in [2]-[5]. It is important to indicate that in these works the analysis is based on two specific assumptions regarding earth conduction effects on propagation characteristics:

- The influence of the imperfect earth is considered only

on the cable impedance; thus, cable admittance earth conduction effects are neglected. This implies that the earth is assumed to act as an electrostatic shield and the accuracy of such approaches is limited to low-frequency applications [6], e.g., up to a few kHz. Many efforts to develop expressions for the series impedances have been reported in literature, with the most known proposed by Pollaczek [7], being implemented in EMT-type simulation tools, and by Sunde [8], that extends [7] by including the influence of earth permittivity.

- The electrical properties of the soil, i.e., conductivity and permittivity are assumed constant; however, in reality they are frequency-dependent (FD) [6], [9]-[23].

In order to develop more accurate earth models, approaches involving earth correction terms for both the impedance and admittance of underground cables have been proposed in [24]-[26], raising the first of the above assumptions.

Additionally, several models have been proposed for the prediction of the FD soil electrical properties [9]-[21], [23], as summarized in [22] and [23]. However, only a few recent studies have used FD soil models to investigate the propagation characteristics and the transient performance of underground cable systems [6], [27], [28], revealing a significant influence especially for short cable lengths [6]. In particular, in [6] guidelines for the accurate evaluation of earth conduction effects on the transient performance of underground multiconductor cable systems have been introduced. However, the analysis has been presented only for cables in flat formation.

This paper extends previous work [6] by investigating the propagation characteristics and the transient responses of different multiconductor, underground cable systems, i.e., flat, vertical and trefoil arrangements. The propagation characteristics of the cable system are calculated by using the FD soil model of [12], considering both the generalized earth formulation of [24] and the approximate earth formulation of Sunde [8]. Results are compared and discussed also on the basis of EM transient responses, demonstrating the applicability of the guidelines introduced in [6] for the investigation and accurate evaluation of earth conduction effects on the transient performance of underground cable systems.

II. GUIDELINES FOR ACCURATE TRANSIENT ANALYSIS

Based on the results and analysis presented in [6], guidelines have been developed for the accurate assessment of earth conduction effects on the propagation characteristics and

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the transient response of underground multiconductor cable systems. Three critical frequencies (f_{c1}, f_{c2}, and f_{c3}) have been introduced to evaluate earth conduction effects: f_{c1} refers to soil modeling, f_{c2} to earth formulation, and f_{c3} to cable transient response. These frequencies can be employed as criteria so as to determine whether to use FD soil modeling and the generalized earth formulation in transient analysis of underground cable systems. The procedure for determining and applying the critical frequency criteria, as well as the proposed modeling approach are concisely summarized here; a more detailed analysis can be found in [6].

A. Soil modeling criterion

The f_{c1} frequency criterion is determined based on the comparison between the FD soil conductivity, σ_{FD}, predicted by the adopted FD soil model, and the low-frequency soil conductivity, $\sigma_{1,LF}$, typically used in modeling approaches with constant (frequency-independent) soil electrical properties. In fact, f_{c1} can be considered as the frequency where $\sigma_{FD}/\sigma_{1,LF}$ becomes equal to 1.1 [6]. For $f>f_{c1}$, FD soil electrical properties should be used in transient analysis of underground cable systems, as their effect on cable propagation characteristics is significant; this is mainly due to the dispersion of soil conductivity especially in poorly conductive soils.

The FD soil electrical properties can be predicted by the Longmire and Smith [12] soil model; the latter was selected based on the discussion on FD soil modeling presented in [6], as well as on the results on cable propagation characteristics of [6]. The Longmire and Smith [12] FD soil model has been derived from laboratory measurements and verified using circuit analysis; it is valid in the frequency range of 100 - $2 \cdot 10^8$ Hz. The relative permittivity, ε_{r1}, and effective conductivity, σ_1 (S/m), are given as

$$\varepsilon_{r1}(f) = \varepsilon_{r1,\infty} + \sum_{n=1}^{13} \frac{a_n}{1+(f/f_n)^2}, \quad (1)$$

$$\sigma_1(f) = \sigma_{1,DC} + 2\pi f \varepsilon_0 \sum_{n=1}^{13} \frac{a_n f/f_n}{1+(f/f_n)^2} \quad (2)$$

where $\varepsilon_{r1,\infty}$ is the high frequency relative permittivity of soil, ($\varepsilon_{r1,\infty} = 5$ [12]), $\sigma_{1,DC}$ (S/m) is the DC soil conductivity, ε_0 is the permittivity of free space ($8.854 \cdot 10^{-12}$ F/m), a_n (pu) are empirical coefficients (Table I), f is the frequency in Hz, and f_n (Hz) are scaling coefficients calculated as [22]

$$f_n = 10^{n-1} \left(125\sigma_{1,DC}\right)^{0.8312}. \quad (3)$$

Note that the Longmire and Smith model can also be considered as appropriate for EMT simulations with $f < 100$ Hz. This is because the predicted decrease of soil conductivity with decreasing frequency is in line with experimental results on actual soils, such as those of [19].

TABLE I
EMPIRICAL COEFFICIENT VALUES FOR THE LS [12] SOIL MODEL

n	1	2	3	4	5	6	7	8	9	10	11	12	13
a_n (pu)	3.40 x10^6	2.74 x10^5	2.58 x10^4	3.38 x10^3	5.26 x10^2	1.33 x10^2	2.72 x10^1	1.25 x10^1	4.80 x10^0	2.17 x10^0	9.80 x10^{-1}	3.92 x10^{-1}	1.73 x10^{-1}

B. Earth formulation criterion

The f_{c2} frequency criterion associated with earth formulation is determined empirically based on the analysis

of [6]. For poorly conductive soils (~0.001 S/m), the order of magnitude of f_{c2} is some hundreds of Hz, whereas for conductive soils (~0.01 S/m) some tens of kHz. For $f>f_{c2}$, the generalized earth formulation [24] should be used considering both earth series impedance and shunt admittance of the cable conductors and sheaths instead of the approximate earth formulation of Sunde [8].

According to the generalized earth formulation of [24] and by considering two single-core cables buried in a homogeneous earth (Fig. 1), the influence of earth conduction effects on the per-unit-length cable parameters is described by the self and mutual earth impedance, Z'_e, and admittance, Y'_e, terms. The general form of these terms is [24]

$$Z'_e = \frac{j\omega\mu_1}{2\pi} \int_0^{+\infty} F_e(\lambda)\cos\left(y_{ij}\lambda\right)d\lambda, \quad (4)$$

$$F_e(\lambda) = \frac{e^{-\alpha_1'|h_i-h_j|} - e^{-\alpha_1'(h_i+h_j)}}{\alpha_1'} + \frac{2\mu_0 e^{-\alpha_1'(h_i+h_j)}}{a_1'\mu_0 + a_0'\mu_1}, \quad (5)$$

$$Y'_e = j\omega P_e^{-1}, \quad (6)$$

$$P_e = \frac{j\omega}{2\pi(\sigma_1 + j\omega\varepsilon_1)} \int_0^{+\infty} \left[F_e(\lambda) + G_e(\lambda)\right]\cos\left(y_{ij}\lambda\right)d\lambda, \quad (7)$$

$$G_e(\lambda) = \frac{2\mu_0\mu_1\alpha_1'\left(\gamma_1^2 - \gamma_0^2\right)e^{-\alpha_1'(h_i+h_j)}}{(a_1'\mu_0 + a_0'\mu_1)(a_1'\gamma_0^2\mu_1 + a_0'\gamma_1^2\mu_0)} \quad (8)$$

where $a_k' = \sqrt{\lambda^2 + \gamma_k^2 + k_x'^2}$ and $k_x' = \omega\sqrt{\mu_1\varepsilon_1}$. ε_0, μ_0, σ_0 are respectively the permittivity, permeability and conductivity of air with $\sigma_0 = 0$. The corresponding earth properties are $\varepsilon_1 = \varepsilon_{r1}\varepsilon_0$, $\mu_1 = \mu_{r1}\mu_0$ and σ_1, where the subscript r denotes relative quantities. The propagation constants for air ($m = 0$) and earth ($m = 1$) are given as

$$\gamma_m = \sqrt{j\omega\mu_m\left(\sigma_m + j\omega\varepsilon_m\right)}. \quad (9)$$

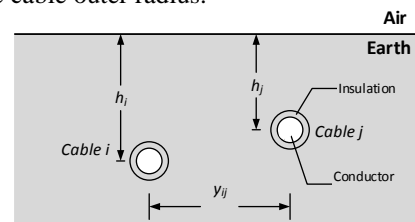Cable self-parameters are derived by considering $h_j = h_i$ and y_{ij} equal to the cable outer radius.

Fig. 1. Layout of two insulated underground cables.

C. Response criterion

The f_{c3} frequency criterion is associated with the length of the underground cable system, as a lower cable length enhances earth conduction effects. f_{c3} is defined as the 1st order cable resonance frequency (RF) of the ground mode [6]; this is the lowest RF of all propagation modes. This frequency criterion can be considered as an upper-frequency boundary.

D. Application of the frequency criteria

Based on the frequency criteria f_{c1}, f_{c2}, and f_{c3}, FD soil modeling, that is, the Longmire and Smith [12] soil model, and the generalized earth formulation of [24], (4)-(9) should be considered when $f_{c3}>f_{c1}$ and $f_{c3}>f_{c2}$, respectively; in these

cases their influence on the transient response is considerable.

III. PROPAGATION CHARACTERISTICS

The propagation characteristics of three identical single-core cables are calculated by using the methodology of [6]. The frequency-dependence of the soil electrical properties is predicted by applying the Longmire and Smith [12] soil model, assuming $\sigma_{1,LF}$ equal to 0.01 S/m and 0.001 S/m to cover soil cases ranging from highly to poorly conductive. The cross-section, EM and geometrical characteristics of the single cable are depicted in Fig. 2. The cables are considered in trefoil, flat and vertical formation as shown in Fig. 3.

Core
$r_c = 11.9$ mm
$\rho_c = 2.1 \cdot 10^{-8}\ \Omega \cdot$m

Insulation
$r_{ins} = 18.55$ mm
$\varepsilon_{r,ins} = 2.72$

Sheath
$r_{sh} = 19.75$ mm
$\rho_{sh} = 3.26 \cdot 10^{-8}\ \Omega \cdot$m

Jacket
$r_{jack} = 22.75$ mm
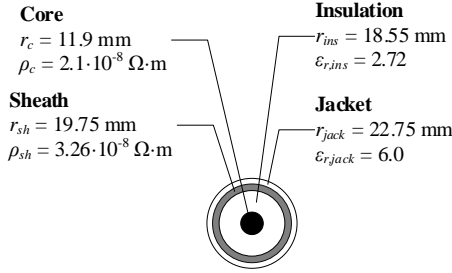$\varepsilon_{r,jack} = 6.0$
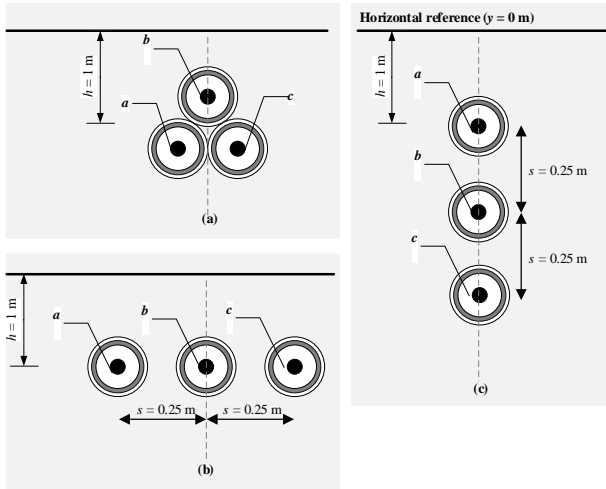
Fig. 2. Single cable configuration.

Fig. 3. Cable systems of a) trefoil, b) flat and c) vertical arrangement.

The propagation characteristics of the cable systems are decomposed to six cable modes, i.e., the ground mode, two inter-sheath modes and three modes of coaxial nature. In particular, the ground mode is a zero-sequence sheath mode of propagation, inter-sheath mode #1 involves wave propagation in all three sheaths, while #2 in the sheaths of cables of phase a and c [2]. This implies that the propagation characteristics of the ground and the two inter-sheath modes are mostly influenced by the soil properties compared to the coaxial modes [6]; thus, these are mainly considered in this analysis. The earth conduction effects are examined, assuming the generalized [24] and the approximate earth formulation of Sunde [8]. Note that, Sunde's earth approach can be derived assuming $k'_x = 0$ in (4)-(8) and, most importantly, considering earth as an electrostatic shield between the cables thus neglecting the earth admittance (6).

The calculated ground and inter-sheath mode #1 attenuation constant and velocity for the two earth approaches are compared in Figs. 4 and 5, respectively; results are for the different cable arrangements and for $\sigma_{1,LF} = 0.01$ S/m. Note that, the propagation characteristics for the two inter-sheath modes are almost similar, thus those of inter-sheath #1 are only presented. It can be seen that the propagation characteristics of the inter-sheath mode for both earth formulation approaches are sensitive to the cable arrangement; this is more pronounced for the mode velocity. Moreover, the velocity of the inter-sheath mode decreases with the cable separation (Fig. 5); this is in consistency with the results of [2] and applies also at low frequencies for the inter-sheath mode attenuation constant. Concerning the latter, at higher frequencies (above ~10 kHz for the results obtained by the generalized approach and above ~300 kHz for the approach of Sunde) a different behavior is observed. For the ground mode, the cable arrangement has a slight influence on the mode velocity. On the other hand, the ground mode attenuation constant is similar for all cable arrangements. In general, the modal propagation characteristics of the flat and the vertical cable arrangements present comparable behavior; considerable differences are observed with the trefoil case.
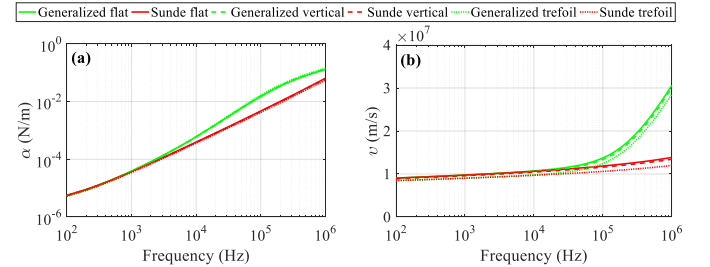

Fig. 4. Ground mode (a) attenuation constant, (b) velocity for $\sigma_{1,LF} = 0.01$ S/m.
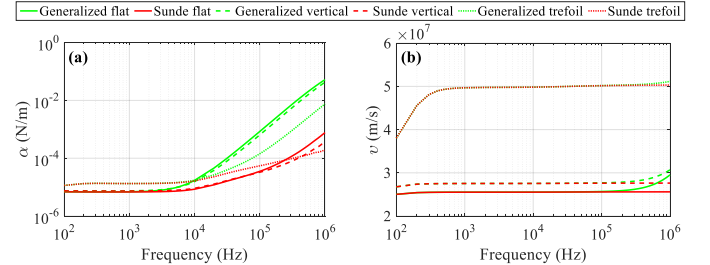

Fig. 5. Inter-sheath mode #1 (a) attenuation constant and (b) velocity, for $\sigma_{1,LF} = 0.01$ S/m.

Comparing the results of the two earth approaches, it can be seen that the modal properties calculated by the generalized approach are frequency-dependent; as expected, this is mostly evident for the attenuation constant [29]. Following Sunde's approach that considers earth as a perfect conductor for the calculation of the cable admittance, the inter-sheath mode #1 velocity is exclusively determined by the EM characteristics of the cable outer sheaths, thus remains constant for frequencies higher than 200 kHz (Fig. 5b). Moreover, combining the results of Figs. 4 and 5, it can be realized that the ground and the inter-sheath propagation constants obtained by the generalized model tend to converge at 1 MHz; this is more pronounced for the flat and the vertical arrangement. Those obtained by Sunde's approach show a significant divergence [29].

To evaluate further the results between the two earth approaches the generalized propagation characteristics were normalized with respect to those obtained using Sunde's approach

$$a_{norm}(f), \; \upsilon_{norm}(f) = \frac{\left| \text{propagation characteristics}_{generalized}(f) \right|}{\left| \text{propagation characteristics}_{Sunde}(f) \right|} \quad (10)$$

where $a_{norm}(f)$ and $\upsilon_{norm}(f)$ is the normalized mode attenuation constant and velocity, respectively. In Figs. 6 and 7 the ratios of the ground mode are plotted for $\sigma_{1,LF}$ equal to 0.01 S/m and 0.001 S/m, respectively. The corresponding plots for the inter-sheath mode #1 are depicted in Figs. 8 and 9.

As expected, according to the f_{c2} criterion the ground mode attenuation constant between the two approaches presents significant differences, starting at ~10 kHz for $\sigma_{1,LF} = 0.01$ S/m and 0.2 kHz for soil case $\sigma_{1,LF} = 0.001$ S/m for all cable arrangements. The differences at low frequencies, where earth behaves mainly as a conductor, are attributed to the influence of the earth conduction current on the cable sheath conductance. As frequency increases, differences are also due to the increasing influence of earth displacement current on the corresponding cable sheath capacitances [6]. The differences in the ground mode characteristics are more evident for the trefoil arrangement.

A significant feature is that the inter-sheath mode attenuation constant between the two approaches presents high differences, thus resulting into $a_{norm}(f)$ exceeding 10, for frequencies starting at ~50 kHz (flat) - 300 kHz (trefoil) for $\sigma_{1,LF} = 0.01$ S/m and 10 kHz (flat) - 60 kHz (trefoil) for $\sigma_{1,LF} = 0.001$ S/m. This is attributed to the fact that $Y_{ij}=G_{ij}+j\omega C_{ij}=0$, since no electrostatic coupling between the cables is considered by following Sunde's approach. For the ground mode differences are significantly lower, since $Y_{ii}=j\omega C_{ii}\neq0$ due to the cable jacket capacitance. In general, it can be deduced that for the inter-sheath mode the differences between the two earth approaches are mostly evident for the flat and the vertical cable arrangements, due to the increased conducting earth path between the adjacent sheaths compared to the trefoil case.
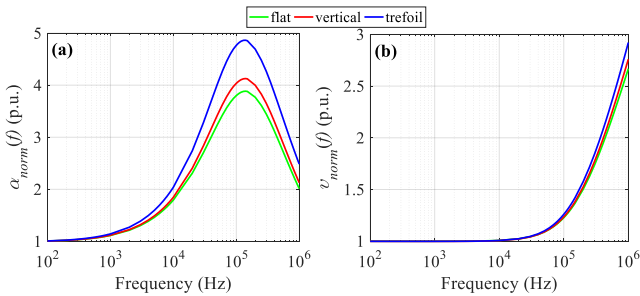

Fig. 6. Normalized ground mode (a) attenuation constant, (b) velocity, for $\sigma_{1,LF} = 0.01$ S/m.
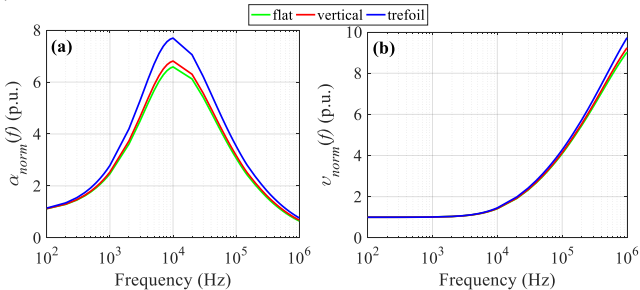

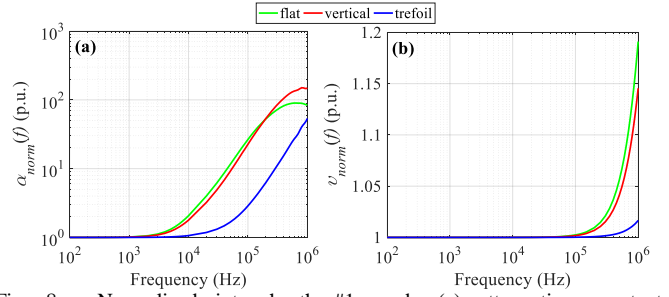Fig. 7. Normalized ground mode (a) attenuation constant, (b) velocity, for $\sigma_{1,LF} = 0.001$ S/m.

Fig. 8. Normalized inter-sheath #1 mode (a) attenuation constant, (b) velocity, for $\sigma_{1,LF} = 0.01$ S/m.
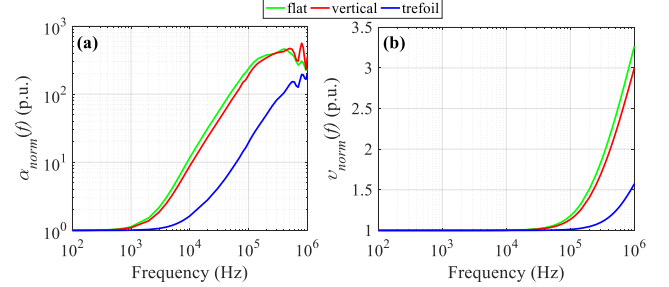

Fig. 9. Normalized inter-sheath #1 mode (a) attenuation constant, (b) velocity, for $\sigma_{1,LF} = 0.001$ S/m.

IV. TRANSIENT RESPONSES

The different earth modeling approaches are also assessed by investigating their impact on the cable transient responses. An ideal 1.2/50 μs double-exponential voltage source is applied to the cable a core sending end. The corresponding receiving end as well as both ends of the remaining cable cores are assumed open-ended. In addition, single-point bonding is applied to cable sheaths with ground resistance of 1 Ω at the sending end. The transient responses are obtained for different cable lengths using the simulation model of [30]. The associated RFs are also calculated based on the methodology of [31].

A. Results for $\sigma_{1,LF} = 0.01$ S/m

In Fig. 10, the transient voltages at the receiving end of cable a sheath are shown for all cable configurations, assuming both earth approaches with $\sigma_{1,LF} = 0.01$ S/m and cable lengths equal to 100 m, 1000 m and 4000 m. The comparison between the results with respect to each cable length and configuration reveals the effect of the adopted earth formulation on the transient responses. Specifically, deviations are observed during the total transient response in terms of attenuation rate and travel time. This is attributed to the fact that the generalized approach takes into account the earth admittance in the calculation of the per-unit-length cable parameters, leading to transient responses with faster attenuation rate and travel time. The differences against Sunde's approach are more marked with decreasing cable length, since the frequency context of the transient response shifts gradually to higher frequencies, where the impact of the earth admittance is more significant (see f_{c3} criterion). This is also verified by the corresponding transient response spectrum analysis in Figs. 11, 12 and 13 for the flat, vertical and trefoil configurations, respectively; the spectra of the transient responses refer to the generalized approach. Note that,

differences on the cable core voltages are negligible, since the cable sheath isolates electrostatically the inner cable part from earth.
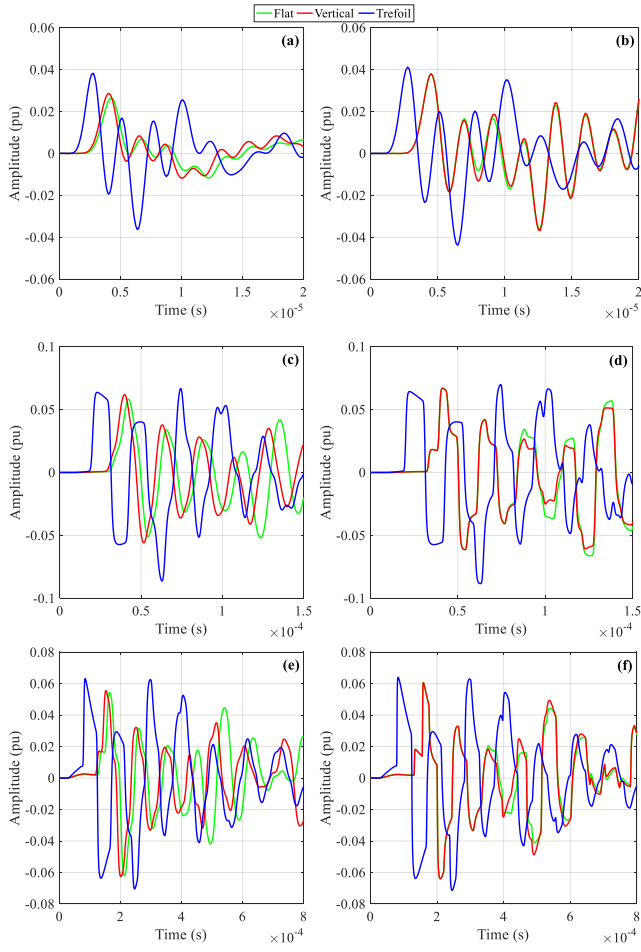

Fig. 10. Transient responses at end R of cable #1 sheath for $\sigma_{1,LF} = 0.01$ S/m, $\ell = 100$ m and (a) Generalized approach, (b) Sunde approach, for $\ell = 1000$ m and (c) Generalized approach, (d) Sunde approach, for $\ell = 4000$ m and (e) Generalized approach, (f) Sunde approach.

Fig. 11. Spectrum of the flat configuration sheath transient response for cable length: (a) 100 m, (b) 1000 m and (c) 4000 m, using the generalized approach.

Fig. 12. Spectrum of the vertical configuration sheath transient response for cable length: (a) 100 m, (b) 1000 m and (c) 4000 m, using the generalized approach.
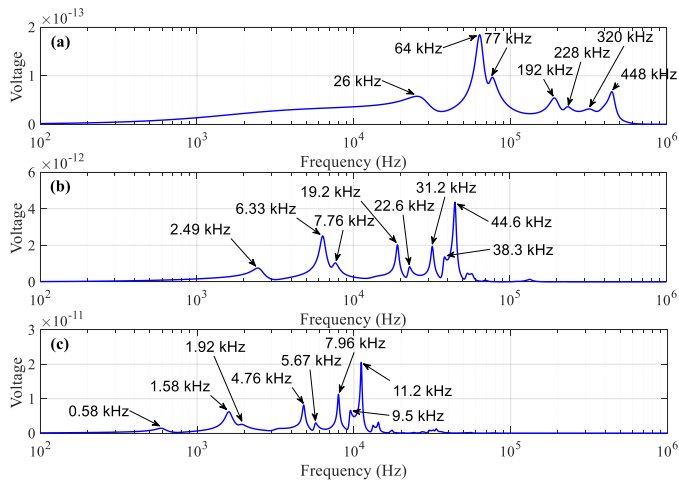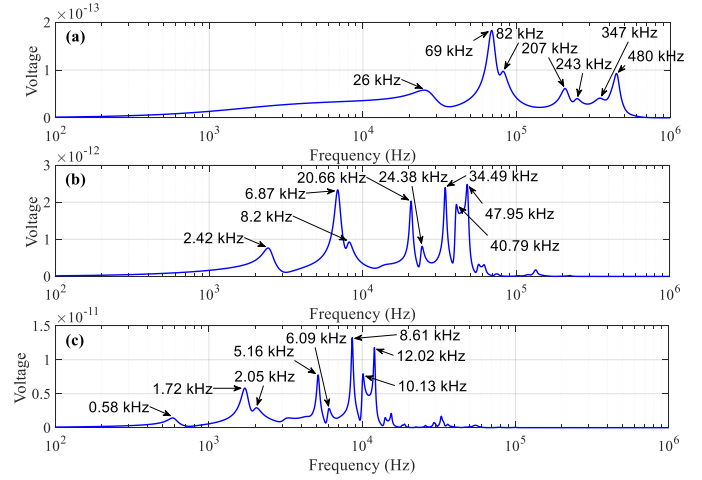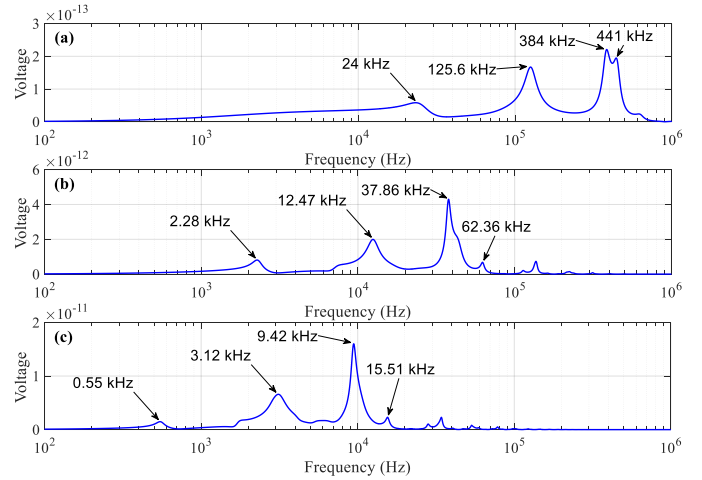

Fig. 13. Spectrum of the trefoil configuration sheath transient response for cable length: (a) 100 m, (b) 1000 m and (c) 4000 m, using the generalized approach.

Regarding the effect of cable configuration with respect to cable length and earth model, the transient responses are generally similar in the cases of flat and vertical configurations, especially at the beginning of the transient. However, differences are observed with time, which stems mainly from the deviations in the inter-sheath mode velocity. Moreover, the transient responses of the trefoil arrangement differ significantly from those of the flat and the vertical configurations. This can be explained through a comparison of their propagation characteristics (see Figs. 4 and 5).

The differences in the waveforms between the examined cable configurations are also directly reflected to the calculated RFs of the transient responses, which are a characteristic of the cable configuration seen as a linear time-invariant system. Results are summarized in Tables II-IV for the flat, vertical and trefoil cable arrangement, respectively, assuming an open-ended cable and thus focusing on the odd multiples of the quarter-wavelength frequency, i.e., from $n = 1$ to $n = 7$. Note that, f_{c3} is the ground mode RF for $n = 1$. As the cable length decreases, the frequency context of the transient responses shift gradually to higher frequencies, where differences in propagation characteristics between the earth

approaches and cable configurations are enhanced. This behavior is also evident to the RFs, which increase as the cable length decreases. For example, for the trefoil arrangement f_{c3} is 25.78 kHz, 2.32 kHz and 0.55 kHz for ℓ equal to 100 m, 1000 m and 4000 m, respectively. In addition, it is noticed that the RFs of flat and vertical configurations yield similar values compared to the corresponding of the trefoil arrangement. This behavior justifies the differences observed in the corresponding transient responses of Fig. 10 between the three cable configurations.

TABLE II
FLAT ARRANGEMENT RFs (kHz)

Cable length (m)	RF order	Cable modes		
		Ground ($r = 0$)	Inter-sheath #1 ($r = 1$)	Inter-sheath #2 ($r = 2$)
100	$n = 1$	28.70	64.05	75.86
	$n = 3$	86.10	192.14	227.59
	$n = 5$	143.51	320.23	379.32
	$n = 7$	200.91	448.32	531.04
1000	$n = 1$	2.52	6.38	7.55
	$n = 3$	7.95	19.16	22.69
	$n = 5$	13.62	31.96	37.86
	$n = 7$	17.67	44.67	52.88
4000	$n = 1$	0.60	1.60	1.89
	$n = 3$	1.87	4.79	5.67
	$n = 5$	3.18	7.98	9.44
	$n = 7$	4.19	11.17	13.21

TABLE III
VERTICAL ARRANGEMENT RFs (kHz)

Cable length (m)	RF order	Cable modes		
		Ground ($r = 0$)	Inter-sheath #1 ($r = 1$)	Inter-sheath #2 ($r = 2$)
100	$n = 1$	28.07	69.09	80.79
	$n = 3$	84.20	207.28	242.36
	$n = 5$	140.33	345.60	403.94
	$n = 7$	196.50	483.64	565.52
1000	$n = 1$	2.48	6.86	8.04
	$n = 3$	7.45	20.66	24.13
	$n = 5$	12.42	34.43	40.22
	$n = 7$	17.38	48.20	56.31
4000	$n = 1$	0.59	1.72	2.01
	$n = 3$	1.77	5.16	6.03
	$n = 5$	2.96	8.61	10.05
	$n = 7$	4.14	12.05	14.07

TABLE IV
TREFOIL ARRANGEMENT RFs (kHz)

Cable length (m)	RF order	Cable modes		
		Ground ($r = 0$)	Inter-sheath #1 ($r = 1$)	Inter-sheath #2 ($r = 2$)
100	$n = 1$	25.78	125.62	125.56
	$n = 3$	77.34	376.87	376.69
	$n = 5$	128.91	628.12	627.81
	$n = 7$	180.47	879.36	878.93
1000	$n = 1$	2.32	12.46	12.47
	$n = 3$	6.95	37.38	37.40
	$n = 5$	11.58	62.30	62.33
	$n = 7$	16.21	87.22	87.26
4000	$n = 1$	0.55	3.11	3.11
	$n = 3$	1.66	9.33	9.34
	$n = 5$	2.77	15.56	15.56
	$n = 7$	3.88	21.78	21.78

B. Results for σ = 0.001 S/m

In Fig. 14, the corresponding transient voltages are summarized for all cable configurations assuming both earth approaches with $\sigma_{1,LF} = 0.001$ S/m and the same cable lengths as in Fig. 10. Transient responses present a similar behavior as for $\sigma_{1,LF} = 0.01$ S/m. However, lower voltage amplitudes are generally observed due to the higher attenuation caused by the lower soil conductivity, as well as higher differences with Sunde's approach (given that for $\sigma_{1,LF} = 0.001$ S/m f_{c2} is lower than for $\sigma_{1,LF} = 0.01$ S/m). In addition, the poorly conductive soil leads to higher deviations between the results of flat and vertical cable configurations. It should be noted that these findings are also justified by the associated RFs, presented indicatively for the flat arrangement in Table V.
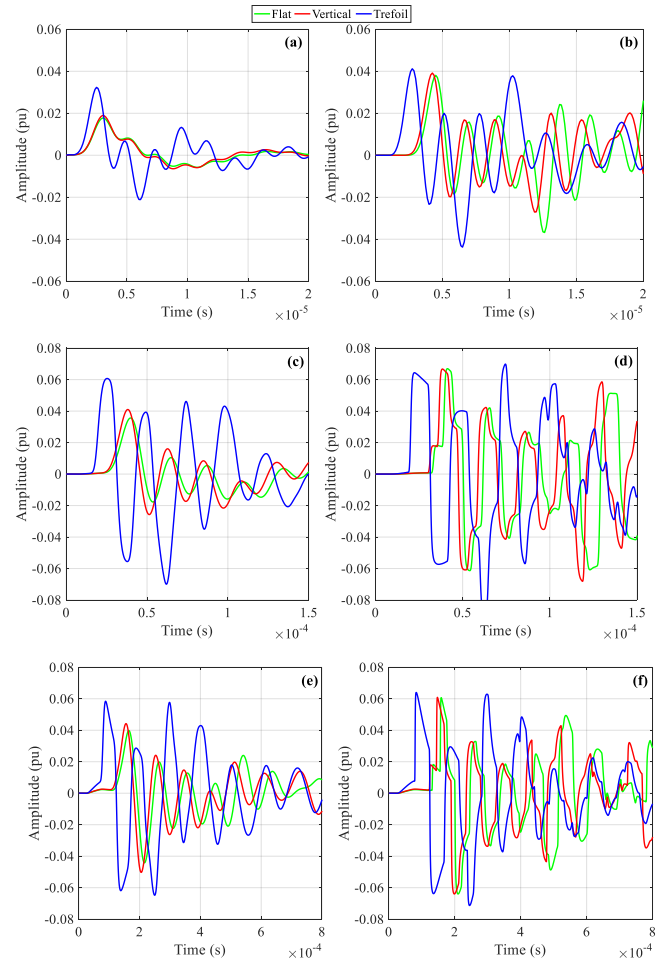

Fig. 14. Transient responses at end R of cable #1 sheath for $\sigma_{1,LF} = 0.001$ S/m, $\ell = 100$ m and (a) Generalized approach, (b) Sunde approach, for $\ell = 1000$ m and (c) Generalized approach, (d) Sunde approach, for $\ell = 4000$ m and (e) Generalized approach, (f) Sunde approach.

Cable length (m)	Cable modes			
	RF order	Ground (r = 0)	Inter-sheath #1 (r = 1)	Inter-sheath #2 (r = 2)
100	n = 1	69.59	69.67	80.64
	n = 3	209.00	208.77	241.91
	n = 5	348.34	347.95	403.19
	n = 7	487.67	487.12	564.46
1000	n = 1	2.41	6.40	7.57
	n = 3	9.01	19.39	22.59
	n = 5	19.82	32.81	38.60
	n = 7	36.40	46.87	54.88
4000	n = 1	0.56	1.60	1.89
	n = 3	1.77	4.79	5.67
	n = 5	3.09	8.00	9.47
	n = 7	4.54	11.23	13.28

V. CONCLUSIONS

The wave propagation characteristics and the transient performance of underground multiconductor cable systems in flat, vertical and trefoil arrangement have been investigated. Computations were performed considering the frequency-dependent (FD) soil electrical properties by adopting the Longmire and Smith soil model, as well as a generalized earth formulation taking into account the impact of earth conduction effects on both series impedance and shunt admittance of the cable conductors and sheaths. Comparisons have been carried out with the approximate earth formulation of Sunde, which neglects the influence of imperfect earth on shunt admittances, thus being applicable to low-frequency range (roughly up to some kHz). Differences are discussed on the basis of previously introduced guidelines for the accurate transient analysis of underground cable systems. These guidelines consider three critical frequency criteria to determine the most accurate modeling approach associated with earth conduction effects in transient analysis of underground cable systems.

Significant differences are found for the ground and inter-sheath modes between the generalized and approximate earth formulations becoming generally more pronounced with increasing frequency and decreasing soil conductivity. For the ground mode the differences are more evident for the trefoil arrangement, whereas for the inter-sheath mode for the flat and the vertical arrangements. The latter is due to the increased conducting earth path between the adjacent sheaths compared to the trefoil case.

The transient responses of flat and vertical cable configurations present similar behavior in general, as a result of their comparable propagation characteristics. Considerable differences are observed for the trefoil arrangement as its propagation characteristics differ significantly. This is reflected to the calculated resonant frequencies (RFs) of the transient responses, a characteristic of the cable configuration. The RF values for the flat and vertical configuration are similar, whereas those corresponding to the trefoil arrangement differ.

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