Analysis of Transient Voltages and Currents in Short Transmission Lines on Frequency-Dependent Soils

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Abstract—Accurate modeling of overhead transmission lines (OHTLs) for transient analysis require that the ground which the phase conductors are suspended be considered on the longitudinal impedance and transversal admittance matrices. In this framework, some models deal with the soil as an ideal conductor (constant resistivity \( \rho_g \) and relative permittivity \( \varepsilon_r \)). However, it is known fact that real soils are characterized by frequency-dependent (FD) electrical parameters \( \rho_g(f) \) and \( \varepsilon_r(f) \). Due to these conditions, different formulations to represent the soil with FD parameters have been developed in the last decades. In order to obtain a precise transient response, these models must be incorporated in longitudinal impedance, as the ground-return impedance, and transversal admittance matrices. In this article, an analysis to compute the impact of some FD soil electrical parameters on transient responses is carried out. These responses are calculated for an energization maneuver and lightning direct strike on OHTLs with different lengths located above constant and FD soil parameter models of low and high soil resistive values. Results show significant differences between the transient responses obtained with the constant and FD soil models, which these variations are more pronounced for soils of high resistivity and for short OHTLs.

Keywords—Electromagnetic transients, frequency-dependent soils, transmission lines, ground-return impedance

I. INTRODUCTION

A CCURATE modeling is required to represent several components over a large frequency content in power systems and to assess the electromagnetic transients adequately. In this framework, OHTLs and underground cables must be modelled from DC up to few tens of MHz, which the longitudinal and the traversal parameters are considerably affected by the ground, assumed as an ideal soil in most approaches [1], [2]. Furthermore, in real ground, the frequency dependence of the soil electrical parameters (resistivity \( \rho_g \) and permittivity \( \varepsilon_r \)) must be considered for soils of moderate and high resistivity [3].

The first approach developed to include the ground effect on the longitudinal impedance (so-called ground-return impedance \( Z_g(\omega) \)) was proposed by Carson [1]. In these expressions, the ground-return impedance is given by improper integrals that can not be analytically integrated in a closed-form solution and approximated formulas based on series and asymptotic expansions for numerical evaluations were established [4]. In these series, the soil resistivity is considered constant and the soil relative permittivity is equal to 1. Later, in 1968, Sunde [2] developed a closed-form expression to determine the ground-return impedance which considers the soil resistivity \( \rho_g \) and relative permittivity \( \varepsilon_r \) (both assumed frequency-constant) with the propagation effects on the soil (further detailed). However, real soils are composed by organic matter, mineral and water content organized in layers of ground and characterized by the resistivity \( \rho_g \), by the relative permittivity \( \varepsilon_r \) and by the permeability, assumed constant and approximated as the vacuum \( \mu \approx \mu_0 \). On the other hand, the \( \rho_g(f) \) and \( \varepsilon_r(f) \) are considerably due to environmental factors and polarization effects on the soil samples [3], [5], [6], [7], [8].

3.2 The FD soil models are important to properly design the electrical supportability of many components such as insulator strings, pre-insertion resistors, circuit breakers and surge arresters. If the voltage peaks obtained for constant soil models are considered, an overestimation on the insulation level of these components may occur. Concerning the transient currents, a correct actuation on protection devices during faults in power system might be also affect. In this case, improper operation may occurs at the protective devices (relays), leading to outages in power systems and deterioration of the energy supplied. Additionally, the Transient Ground Potential Rise (TGPR) in grounding systems, the lightning radiated electromagnetic fields and induced voltages are significantly affected by the FD soil models, specially in high-resistive soils as shown in [7], [9], [10], [11].

In order to evaluate the impact of FD soils, a comparison between the transient responses computed considering the constant and FD soil models are presented. For this study, two OHTLs of 0.5 km and 5 km located above soils of 500 and 2,500 \( \Omega \cdot \text{m} \) are considered. Then, these OHTLs are subjected to different scenarios (energization maneuver and lightning strike) where the voltages for open-circuit and short-circuit currents are computed. Results show relevant differences in harmonic impedances of these OHTLs between the FD models employed and that behaviour reflects in the transient voltage and current responses which these variations are more pronounced in short OHTLs over high-resistive soils.

II. TRANSMISSION LINE MODELLING

Assuming that a single-phase OHTL of length \( d \) and radius \( r \) is parallel to the ground which a real soil is represented by...
by a magnetic permeability ($\mu_0$), a relative permittivity $\varepsilon_r(f)$ and a resistivity $\rho(f)$ as illustrated in Fig.1-(a). The voltage (V) and current (I) along the x-axis are computed, in frequency domain, as follows [12]

$$\frac{dV(\omega)}{dx} = -[Z]\mathbf{I}(\omega) = -[Z'(\omega) + Z_e(\omega) + Z_g(\omega)]\mathbf{I}(\omega)$$

$$\frac{dI(\omega)}{dx} = -[Y]\mathbf{V}(\omega) = -\left[Y^{-1}(\omega) + Y_e^{-1}(\omega)\right]^{-1}\mathbf{V}(\omega)$$

where $Z'$ is the longitudinal impedance and $Y'$ is the transversal admittance, in per unit length (p.u.l.), for a differential length $dx$. The longitudinal impedance is composed by the sum of the internal impedance $Z'_e$, due to the skin effect, by the external impedance $Z'_g$, due to the external magnetic field to other conductors and by the ground-return impedance $Z'_r$ due to magnetic field that penetrates the soil. The transversal admittance $Y'$ is composed by the external admittance $Y'_e$, computed for an ideal soil (perfect conductor) and the admittance $Y'_g$ is a correction term for real soils [12]. The p.u.l. equivalent circuit is depicted in Fig.1-(b).

Several approaches have been proposed to calculate the admittance and impedance of the ground ($Z'_r$, $Y'_r$), which $Y'_g$ can be neglected without significant errors [13]. On the other hand, ground-return impedance plays a fundamental role due its high contribution to the longitudinal impedance $Z'$. In this context, Carson [1] investigated the soil effect on OHTLs and established general solutions based on Bessel and Struve functions. Due its complexities, a series expansion was presented by Carson and is incorporated in EMTP-tool programs [14]. Decades later, other authors have proposed approximated equations based on the complex depth known as closed form expressions [2], [15]. Sunde proposed equations for the proper terms [2] which later they were extended by Rachidi [15] to include the mutual terms. These equations consider the phase conductors of infinite length functioning located above a perfect soil and take into account the influence of the displacement current on the soil, with the soil permittivity with the term $j\omega\varepsilon_g$, where $\varepsilon_g = \varepsilon_r\varepsilon_0$ and are valid for conductivity soils up to 0.0001 S/m considering OHTLs with short lengths [16]. These expressions are given by

$$Z_{e0} = j\omega\mu_0\frac{1}{2\pi} \ln \left[\frac{1 + \gamma g h_i}{\gamma g h_i}\right]$$

$$Z_{r0} = j\omega\mu_0\frac{1}{4\pi} \ln \left[\frac{1 + 0.5\gamma g (h_i + h) + (0.5\gamma g r_i)^2}{0.5\gamma g (h_i + h)^2 + (0.5\gamma g r_i)^2}\right]$$

where the angular frequency is $\omega = 2\pi f$ [rad/s], the frequency is $f$ [Hz], the vacuum magnetic permeability is $\mu_0 = 4\pi \times 10^{-7}$ H/m, the conductor’s height above the soil are $h_i$ and $h$ [m], the distance between the conductors is $r_{ij}$ [m], the soil conductivity is $\sigma_g$ [S/m], the vacuum permittivity is $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m and the relative permittivity is $\varepsilon_r$. Sunde’s closed expression adapted by replacing $\sigma_g$ for $\sigma_g(f)$ and $\varepsilon_r$ for $\varepsilon_r(f)$ makes possible to include the FD soil models in the ground-return impedance.

### III. Frequency-Dependent Soil Models

Many authors have proposed different formulations based on sample measurements in field and in laboratory to consider the FD on the soil parameters as e.g. [3], [6], [7], [17]. The variation that occurs in the soil electrical parameters may significantly affect the transient responses, specially for high-frequency phenomena such as lightning. Four of these formulations are described below.

#### A. Visacro and Portela (VP)

The soil model given by Visacro and Portela [17] in 1987 is based on laboratory tests with samples from three different soils. As a result, the researchers proposed empirical formulations reproducing the variation of the soil conductivity ($\sigma_g(f)$) and the relative permittivity ($\varepsilon_r(f)$) which are given by

$$\sigma_g(f) = \sigma_0(f - 100)^{0.072}$$

$$\varepsilon_r(f) = 2.34 \times 10^6 (1/\sigma_0^{0.535} f^{-0.597})$$

where $\sigma_0$ is the conductivity at low frequency measured at 100 Hz. The expressions are valid the frequency range from 40 Hz up to 2 MHz.

#### B. Portela (P)

In 1999, Portela [6] developed a model using soil samples measured in the frequency range of 100 Hz to 2 MHz obtained in different areas of Brazil ranging from rocks to sand and pulverulent soils. From these samples, the following expressions for the calculation of $\sigma_g(f)$ and $\varepsilon_r(f)$ are established, given by

$$\sigma_g(f) = \sigma_0 + \Delta_1 \left[\cot \left(\frac{\pi}{2} \alpha\right)\right] \left(\frac{f}{10^6}\right)^{\alpha}$$

$$\varepsilon_r(f) = \Delta_1 \left(\frac{f}{10^6}\right)^{\alpha} \frac{1}{2\pi f \varepsilon_0}$$
where \( \Delta_t = 2\pi f \varepsilon \) is computed at 1 MHz and depends on the soil model and \( \alpha \) is an adjustable soil parameter. The median values of \( \Delta_t = 11.71 \text{ mS/m} \) and of \( \alpha = 0.706 \) were assumed, based on [10].

### C. Alípio and Visacro (AV)

Alípio and Visacro [3] developed in 2014 a semi-theoretical causal model that describes frequency dependence on soil parameters \( (\sigma_t(f) \text{ and } \varepsilon_t(f)) \). The expressions were obtained based on a data set of field measurements in different locations in Brazil and also on fundamental electromagnetic principles, notably, Maxwell’s equations and the Kramers-Kronig relations and are written as

\[
\sigma_t(f) = \sigma_0 + \sigma_0 \times 1.26\sigma_0^{-0.73} \left( \frac{f}{1\text{MHz}} \right)^\xi \tag{10}
\]

\[
\varepsilon_t(f) = \varepsilon_\infty + \frac{\tan(\pi \xi/2)}{2\pi \varepsilon_0(1\text{MHz})^\xi} \sigma_0 \times 1.26\sigma_0^{-0.73} f^\xi \tag{11}
\]

where \( \varepsilon_\infty = 12 \) and \( \xi = 0.54 \). The frequency range is valid from 100 Hz up to 4 MHz.

### D. CIGRE (C)

Recently, the CIGRE Work Group C4.33 [7] have proposed a formulation that express a causal and a general relation to predict the variation of soil parameters with the frequency \( (\sigma_t(f) \text{ and } \varepsilon_t(f)) \), which are also a function of low-frequency soil conductivity \( (\sigma_0) \). These expressions are written as

\[
\sigma_t(f) = \sigma_0 + 4.7 \times 10^{-6} \sigma_0^{0.27} f^{0.54} \tag{12}
\]

\[
\varepsilon_t(f) = 12 + 9.5 \times 10^4 \sigma_0^{0.27} f^{-0.46} \tag{13}
\]

All FD models will be used to compute the transient responses on OHTLs under different scenarios.

### IV. Numerical results

The results are organized in two sections: In section IV-A, the frequency domain responses of the harmonic impedances are computed for two types of OHTLs located above two homogeneous soils. Then, in section IV-B, the transient responses are computed in three different scenarios. In this analysis, the tower impedance and the soil ionization were neglected.

#### A. Frequency-domain responses

In order to evaluate the impact of constant and and FD soil models, the harmonic impedances of two open-circuit single-phase OHTLs are computed for both approaches. The harmonic impedance is given by

\[
Z_h(\omega) = Z_C(\omega) \coth (\gamma(\omega) d) \tag{14}
\]

where \( Z_C(\omega) \) and \( \gamma(\omega) \) are the characteristic impedance and propagation function and are given by

\[
Z_C(\omega) = \sqrt{Z'(\omega)/Y'(\omega)} \quad \gamma(\omega) = \sqrt{Z'(\omega)Y'\omega} \tag{15}
\]

where \( Z'(\omega) \) and \( Y'(\omega) \) are the longitudinal impedance and transversal admittance of the OHTL and \( d \) is the line length. In these simulations, OHTLs of 0.5-km and 5-km in length, with height of 20 m and the radius of 7.5 mm are employed whose the profile is shown in Fig. 2-(a). These OHTLs are on two types of homogeneous grounds of 500 and 2,500 \( \Omega\cdot\text{m} \), corresponding to a low and a high soil resistivity, respectively [7] and the harmonic impedances are computed by three different approaches: (i) Carson’s formula (Car) using the constant soil resistivity (500 and 2,500 \( \Omega\cdot\text{m} \)) and \( \varepsilon_r = 1 \); (ii) constant soil model with Sunde’s formulas (S) Eqs. ((3)-(4)) with soils of 500 and 2,500 \( \Omega\cdot\text{m} \) and \( \varepsilon_r = 40 \); (iii) Sunde’s formulas with FD soils models \( (\sigma_t(f) \text{ and } \varepsilon_t(f)) \) proposed by Visacro and Portela (VP), Portela (P), Alípio and Visacro (AV) and CIGRE (C). In this case, the low-frequency resistivity \( (\rho_0) \) of 500 and 2,500 \( \Omega\cdot\text{m} \) are considered. The calculated harmonic impedances by these three approaches are shown in Figs. 3 and 4.

It can be seen that the magnitudes of harmonic impedances are in a good agreement at the frequencies corresponding to the first notch (related to the inverse of length \( d \)) and peak. However, as the frequencies increases, the magnitudes for

![Fig. 2: (a) Studied OHTL profile; (b) Step energization; (c) Lightning direct strike.](image)

![Fig. 3: Magnitude of the harmonic impedance \( Z_h(\omega) \) of the 0.5-km OHTL on a soil of: (a) 500 \( \Omega\cdot\text{m} \); (b) 2,500 \( \Omega\cdot\text{m} \).](image)
FD soil models $\sigma_f(f) - \varepsilon_f(f)$ have presented more pronounced amplitudes and shift in comparison with constant soil model Carson (Car) and Sunde (S). The Visacro-Portela (VP) and Portela (P) models have presented the most divergent behaviours in comparison with the other models. Based on these characteristics, the transient responses for OHTLs on FD soil models will be more pronounced for disturbances of high-frequency content (lightning) as described as follow.

**B. Time-domain transient responses with FD models**

In order to investigate the impact of the FD soil models previously studied, the transient responses (voltages and currents) are computed for the OHTLs in this section. For these computations, the same single-phase OHTLs are subjected to two different scenarios: (1) The switching maneuver (energization) which an ideal 1-p.u. step voltage source is applied at the sending end and a load is connected at receiving end as illustrated in (Fig. 2-(b)). Then, transient voltages $V_m(t)$ for the open-circuit and transient currents $I_m(t)$ for the short-circuited are computed; (2) A lightning direct strike hits at the sending end of the OHTL which the sending end is open-circuit as illustrated in Fig. 2-(c). Then, transient voltages $V_m(t)$ are computed in this condition.

The ground-return impedances are calculated using the Sunde’s formulas ((3)-(4)) for the constant and FD soil models and the ground admittance is neglected in these simulations.

All responses are calculated by the Numerical Inverse Laplace Transform method, where the transient voltage $V_m(t)$ and transient current $I_m(t)$ waveforms are depicted in Fig. 5 and in Fig. 6, respectively.

It can be noted that the transient voltages of the Fig. 5 present a damped oscillatory behavior in all cases, where the more pronounced differences are observed for the Portela’s model (P), especially for soils of high resistive value, that presents the highest peaks which are shifted in comparison with the other responses. In order to quantify these differences in the voltage peaks, the percentage variation between the voltage peaks obtained by Sunde’s model (S) and the other ones are calculated. The reference for the voltage occurs at the $7^{th}$ peak and these percentage variations are organized in Table I. As seen, the Portela’s model (P) has presented the highest percentage variation in all cases studied.

In the transient responses for the short-circuit currents of the Fig. 6, the percentage variation $\Delta_1$ is computed between the Sunde’s model (S) and the Visacro-Portela’s model (VP) and $\Delta_2$ is calculated between (S) and Portela’s model (P). These results are shown in the Fig. 6. The highest difference is observed between the constant and FD models are founding for the Portela’s model (P) for a soil of 2,500 $\Omega\cdot$m. As the line length increases, all the transient currents match up, showing that the effects of FD soil models are more pronounced in short lines.

### 2.1 steady-state and current amplitudes

$$i(t) = k_1 e^{-k_2(t-t_0)}; t_0 = \frac{3.5 \times \log(4)}{f_{\text{max}}}$$

where $t_0$ is the delay time, $f_{\text{max}}$ is the frequency decay to half of magnitude of I(s) and $k_1$, $k_2$ are constants. For these simulations, the values of $t_0$ = 1.5444×10⁶ s, $f_{\text{max}}$ = 1 MHz, $k_1$ = $9.6925 \times 10^{-4}$A and $k_2$ = $3.5115 \times 10^{-14}$s⁻² are adopted. The waveforms of the time function I(t) and its Laplace transform are shown (in detail) in Fig. 7-(a) and (b), respectively. For this scenario, only the transient voltages at the receiving end with 0.5-km OHTL on the two types of soils are simulated due to more significant variations in previous scenario. The simulated results are depicted in Fig. 7. As can be observed, the voltages peaks vary for different FD soil models, where Portela’s model (P) have presented highest variation. A comparison between (P) and Sunde’s approach (S) results in 19.10% and 50.24% for the soils of 500 and 2,500 $\Omega\cdot$m, respectively, at the 4th voltage peak. Furthermore, these peaks do not occur at the same time when FD models are considered.

Most of the OHTL models available in the EMTP-type programs consider constant soil models for $\rho_g$ and $\varepsilon_f$ based on the Carson’s and Sunde’s approach. As demonstrated, the FD soil models must be used for a precise transient responses which presents significant differences in comparison with the soil constant model.

### Table I: Variations (%) for the FD soil models studied.

<table>
<thead>
<tr>
<th></th>
<th>$d = 0.5$ km</th>
<th>$d = 5$ km</th>
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<tbody>
<tr>
<td>$\rho_g = 500 \Omega\cdot$m</td>
<td>$\rho_g = 2,500 \Omega\cdot$m</td>
<td>$\rho_g = 500 \Omega\cdot$m</td>
</tr>
<tr>
<td>VP</td>
<td>2.84</td>
<td>3.80</td>
</tr>
<tr>
<td>P</td>
<td>0.20</td>
<td>7.53</td>
</tr>
<tr>
<td>AV</td>
<td>0.26</td>
<td>1.01</td>
</tr>
<tr>
<td>C</td>
<td>0.27</td>
<td>1.02</td>
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are modified in the transient state in these simulations, which may impact the operation of protection devices in power systems if these FD models are used. 1.3 Additionally, the grounding system can be inserted in further analysis in combination a numerical method to compute the grounding system impedance. Then, fitting approaches, such as Vector Fitting technique, can be used to synthesize an equivalent circuit which is incorporate to the EMTP-type programs as an example in [18]. 2.2 As shown in [7], the FD soils must be taken into account for soils with a resistivity higher than 700 $\Omega$.m for transient analysis with OHTLs. As a general recommendation for practical engineering study cases, the

Fig. 5: Transient voltages at the sending end of the 0.5-km OHTL (left column) and 5-km OHTL (right column) over a soil with resistivity $\rho_0$ of: [(a) 500 $\Omega$.m and (b) 2,500 $\Omega$.m].

Fig. 6: Transient currents at the sending end of the 0.5-km OHTL (left column) and 5-km OHTL (right column) over a soil with resistivity $\rho_0$ of: [(a) 500 $\Omega$.m and (b) 2,500 $\Omega$.m].
equations (12) and (13) have presented conservative results with simple implementation into the Sunde’s approach to compute the ground-return impedance. Additionally, these proposed formulations have shown a good agreement in comparison with the experimental data in the literature.

V. CONCLUSIONS

A comparative analysis was carried out in the transient responses for short OHTLs located above grounds represented by constant and FD soil models using the Sunde’s and Carson’s approaches. The harmonic impedances of short OHTLs were computed for two lines located above grounds of low and high resistivity soils, including 4 different FD soil models. The harmonic impedances have present a divergent behaviour as the frequency increases which Portela’s model has shown the highest variations. Transient responses were calculated for an energization maneuver and for a lightning direct strike in these OHTLs. The responses from Portela’s model and Visacro-Portela’s model have presented the highest differences, especially for high resistive soil with lightning strike due to its high frequency content, in comparison with the other FD soil models and constant soil model. The FD soil models must be included for a precise computation leading to significant differences in comparison with the soil constant approach. Furthermore, the voltage peaks do not occur at the same time for some FD models in comparison with ones obtained by the constant soil models. As a general recommendation in practical engineering cases, the formulas proposed by CIGRE must be applied for soils with a resistivity higher than 700 Ω.m for transient analysis with OHTLs instead of constant soil models.

REFERENCES