# Shifted Frequency Analysis-EMTP Multirate Simulation of Power Systems

J. O. Tarazona, A. T. J. Martí, J. R. Martí, F. A. Moreira

***Abstract*-- This paper presents a novel hybrid multirate protocol to interface a Shifted Frequency Analysis (SFA) simulator and an Electromagnetic Transients (EMT) simulator. The proposed protocol is based on the Multi-Area Thévenin Equivalent conceptual framework and uses a second parallel simulation of the EMT simulator to track the imaginary part of the complex-valued solution from the SFA simulator. Having a Real-Part EMT simulation and an Imaginary-Part EMT simulation, running in parallel, maintains the analyticity and synchronicity of the SFA/EMT hybrid simulator and enables the use of large time steps in the SFA solution. This also allows the extraction of the fundamental frequency phasor of the EMT solution directly through the SFA transformation. The interfacing algorithm is demonstrated on the IEEE 39-bus system. The proposed protocol shows very good accuracy for capturing the electromechanical transients with the SFA simulator and an EMT-comparable accuracy with the EMT simulator. The results show the capability of the hybrid simulator to use significantly different solution steps in the SFA and EMT simulators, which results in considerable savings compared with an all-EMT solution. The proposed simulator would be very useful in conducting transient stability-type of studies when there is a need to model some devices with EMT accuracy.**

***Keywords*: Shifted frequency analysis (SFA), Time-varying phasors, Electromagnetic transients (EMT), Power system stability, Hybrid simulation, Multirate simulation.**

## I. Introduction

COMPUTATIONAL tools for transient stability (TS) and electromagnetic transients (EMT) studies have traditionally been used separately. However, with the increase in the complexity of power systems due to the introduction of new technologies and their controls, and an increasing number of interconnections, the simulation needs are more complicated. It was with the advent of the High Voltage Direct Current (HVDC) transmission, that the need first arose to incorporate in the traditional set of tools used for stability studies, the kind of models available in the EMT-type of programs.

The detailed modelling of an HVDC link within a power system stability simulator was first proposed and implemented by Hefferman et al. [1]. In the past four decades, several publications have appeared in the literature for implementing TS-EMT hybrid simulators [2]-[8]. In many hybrid TS-EMT techniques the exchange of information between simulators occurs every TS time step, so at some point in the simulation, the EMT solution would be using a phasor solution that is one TS time step delayed. This may cause inaccuracies in the EMT solution in the presence of disturbances. To correct this, in [5] the interaction algorithm allows the EMT and TS solutions to iterate with each other until convergence is reached, and in [6], the TS and EMT solutions are made consistent by solving the TS and EMT equations simultaneously in a single large system of equations. In traditional hybrid TS-EMT techniques, the TS subsystem is represented in the EMT subsystem as either a multi- or single-port Norton [8] or Thévenin equivalent [3], [4], [5], [7]. For these equivalents, the impedances are evaluated at one single frequency, either the fundamental or the actual system frequency, as in [3] and [5], hence, in the presence of significant transients flowing through the boundaries this representation of the TS subsystem may not be accurate enough for the EMT solution. To solve this problem, in [6] a frequency-dependent network equivalent (FDNE) is used in the Thévenin equivalent of the TS subsystem. However, using an FDNE may cause the solution to become numerically unstable due to the high order transfer function required and, also, any change in the topology of the TS subsystem cannot be easily accommodated because the FDNE is fixed for the whole simulation [8].

Other research in hybrid simulators has centred on developing an integrative approach to model TS and EMT tasks. Instead of coupling two different simulation frameworks, the solution is based on the concept called Frequency Adaptive Simulation of Transients (FAST), to cover the application spectrum of typical EMT and TS programs [9]. In this approach, the entire system is first solved for fast transients in EMT mode with a very small time-step size, which is not very efficient, and then solved for the electromechanical transients in TS mode with a much larger time step size.

The Shifted Frequency Analysis (SFA) method [10] is a time-varying phasor (TVP) approach that is particularly efficient for capturing the oscillations associated with electromechanical transients around the fundamental frequency of the system (50/60 Hz). This is the method used in the TS mode of the FAST approach in [9]. In [11] a transient stability simulator based on SFA was developed.

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SFA is based on shifting by 50/60 Hz the frequency spectrum of the system variables using a time-domain rotational transformation. After shifting, the fundamental 50/60 Hz component of the system signals is centred around zero frequency, thereby allowing the use of large time steps to trace the envelopes of the 50/60 Hz and other waveforms around it, without compromising the accuracy of the solution. The transformed variables in the SFA domain correspond to the conventional power system phasor solutions in magnitude and phase angle. The difference is that in SFA the magnitude and angle of these phasors are functions of time [10]. These time dependencies can be modelled using the traditional EMTP [12] discretization rules, for example, trapezoidal or backward Euler rules. The trapezoidal or backward Euler solutions guarantee absolute numerical stability with a fixed time step. A fixed time step facilitates multirate solutions, where some system regions are naturally slow (e.g., traditional transmission system) and other regions or components are naturally fast (e.g., power electronic components).

In [13] and [14] new hybrid simulation techniques to interface dynamic phasors and EMT simulators were presented. In [13] a dynamic phasor simulator of the generalized averaging method-type [15] was used and in [14] a matrix-transformation-based shifted frequency phasor simulator was employed. Both techniques require transmission lines to decouple the subsystems, which imposes an important restriction because the maximum time step that can be used in the dynamic phasor solution is limited by the travel time of the line.

In this paper, a novel approach is presented whereby an SFA simulation framework for a subsystem is interfaced with an EMT simulation framework for another subsystem. In this hybrid environment, SFA is used to solve for slow electrical and electromechanical transients around the fundamental 60 Hz synchronous frequency, while EMT modelling is used for loads, components, or subsystems that require a much smaller time step for their fast transients. Interfacing of the SFA and EMT simulators is done using the Multi-Area Thévenin Equivalent (MATE) solution framework [16].

The proposed multirate solution guarantees numerical stability at fixed time steps since the two solutions are directly coupled without a time delay: at each EMT time-step, there is available an interpolated value of the SFA solution. This approach increases the accuracy of the hybrid solution and reduces the possibility of numerical instability. In the proposed method, the SFA subsystem is represented, using MATE, as a multi-port Thévenin equivalent that results from the discretization of the differential equations and therefore, can capture the frequency response of the SFA subsystem for a range of frequencies determined by the time step used in the SFA solution. This allows the simulation of scenarios involving harmonic and sub-harmonic resonances and is also important in stability studies with large frequency excursions.

The rest of the paper is structured as follows: in Part II, the SFA technique is presented. The MATE solution framework is reviewed in Part III. The interaction protocol SFA–EMT is described in Part IV. Then, Part V discusses a simulation case study, and in Part VI conclusions and directions for future work are presented.

## II. Shifted Frequency Analysis of Power Systems

A power system bandpass signal $u(t)$, with its frequency spectrum centred on the system frequency $\omega_0$, can be represented as

$$u(t) = u_I(t) cos\omega_0 t - u_Q(t) sin\omega_0 t \tag{1}$$

where the low-pass signals $u_I(t)$ and $u_Q(t)$ are, respectively, the in-phase and quadrature components of $u(t)$. The dynamic or time-varying phasor $\overline{U}(t)$ for the signal $u(t)$ is defined as [17]

$$\overline{U}(t) = u_I(t) + ju_Q(t) \tag{2}$$

$\overline{U}(t)$, known as the complex envelope of $u(t)$ in signal processing, is an analogous complex-valued low-pass representation of the original bandpass signal $u(t)$.

Another representation of $u(t)$ is through its analytic signal $\bar{z}(t)$, defined as

$$\bar{z}(t) = u(t) + jH[u(t)] \tag{3}$$

where H[•] denotes the Hilbert transform [18].

Substituting (1) and (2) in (3) yields

$$\bar{z}(t) = \left(u_I(t) + ju_Q(t)\right) e^{j\omega_0 t} = \overline{U}(t)\, e^{j\omega_0 t} \tag{4}$$

Thus, the analytic signal $\bar{z}(t)$ is a complex-valued representation of the original real-valued signal $u(t)$. Equation (4) shows that a dynamic phasor can be obtained by multiplying the analytic signal by $e^{-j\omega_0 t}$. This can be understood as the analytic signal being mapped into the dynamic phasor through a transformation $T$ defined as follows:

$$T^{-1} = e^{-j\omega_0 t} \tag{5}$$

The signal in the original or "normal" domain is related to the signal in the transformed or "modal domain" (denoted with subscript "m") as follows:

$$\bar{z}_m(t) = T^{-1}\bar{z}(t) \tag{6}$$

Multiplying a signal by $e^{-j\omega_0 t}$ causes its Fourier transform to be shifted $\omega_0$ frequency units to the left. Thus, for a bandpass signal with a frequency spectrum centred on $\omega_0$, the transformed signal is a low-pass signal with its frequency spectrum centred at zero frequency, and, therefore, larger time steps can be used to capture this signal in the simulation. The above transformation is the one used for Shifted Frequency Analysis (SFA) [10] and the transformed signals in the shifted frequency domain or SFA domain, are the corresponding TVP's of the original real-valued time signals.

Using the SFA transformation, EMTP-like equivalent circuits can be derived for the different circuit elements [19]. SFA models have been developed for transformers and transmission lines [10], synchronous machines [20] and induction machines [21]. In [10] a power system transient stability simulator based on the SFA algorithm was developed.

To analyze a power system with SFA, the corresponding analytic signals of the variables of the system at the initial state are obtained and then transformed into the SFA domain. The solution of the circuit in discrete-time is then found by representing each element in the circuit by its SFA model and solving the corresponding system equations. The solution algorithm is the same as for the EMTP [12] except that in SFA the calculations are carried out using complex arithmetic. The solution of the circuit includes the magnitudes and angles of the time-varying phasors in the system [19], which are the envelopes of the real signals in the system. The SFA solution can then be transformed back to normal time domain to get the actual real-valued solution of the system by keeping only the real part of the complex analytic signals.

## III. MATE Conceptual Framework

A detailed presentation of MATE and its formulation in EMTP solutions is available in [16]. Here, a summary of the procedure is presented to establish a frame of reference for the discussions that follow. The MATE solution framework provides an effective means for partitioning a large system into subsystems connected by link branches. The subsystems are solved independently and the whole system solution is found by solving for the links' interactions. In the general formulation of MATE, the link branches can be any type of element, but for the application in this paper, they are considered as resistances with no mutual coupling. The number of subsystems in MATE can be N in general, however, to explain the concept, the system in Fig. 1 composed of two subsystems will be considered.

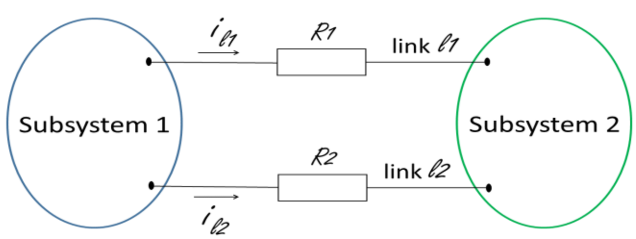


Fig. 1. Sample system for MATE explanation.

The system in Fig. 1 can be described by a set of modified nodal equations [22], as follows:

$$\begin{bmatrix} \boldsymbol{Y}_1 & \boldsymbol{0} & \boldsymbol{C}_1 \\ \boldsymbol{0} & \boldsymbol{Y}_2 & \boldsymbol{C}_2 \\ \boldsymbol{C}_1^T & \boldsymbol{C}_2^T & -\boldsymbol{R}_L \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_1 \\ \boldsymbol{V}_2 \\ \boldsymbol{I}_L \end{bmatrix} = \begin{bmatrix} \boldsymbol{h}_1 \\ \boldsymbol{h}_2 \\ \boldsymbol{0} \end{bmatrix} \tag{7}$$

In MATE, $\mathbf{Y}_j$, $\mathbf{V}_j$ and $\mathbf{h}_j$ are, respectively, for subsystem j, the admittance matrix, the vector of nodal voltages and the vector of injected currents, which includes the history sources in the EMTP model, and the actual sources, but not the currents from the links [16]. Matrix $\mathbf{C}_j$ is the incidence or connection matrix that captures the connectivity of the nodes of subsystem j to the links. $\mathbf{I}_L$ is the vector of link currents and $\mathbf{R}_L$ is a diagonal matrix with the link resistances. Operating on (7) to find $\mathbf{I}_L$ renders

$$\begin{bmatrix} \boldsymbol{1} & \boldsymbol{0} & \boldsymbol{Y}_1^{-1}\boldsymbol{C}_1 \\ \boldsymbol{0} & \boldsymbol{1} & \boldsymbol{Y}_2^{-1}\boldsymbol{C}_2 \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{Z}_T \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_1 \\ \boldsymbol{V}_2 \\ \boldsymbol{I}_L \end{bmatrix} = \begin{bmatrix} \boldsymbol{e}_1 \\ \boldsymbol{e}_2 \\ \boldsymbol{E}_L \end{bmatrix} \tag{8}$$

where,

$$\boldsymbol{e}_j = \boldsymbol{Y}_j^{-1}\boldsymbol{h}_j \tag{9}$$

$$\boldsymbol{e}_{Th,j} = \boldsymbol{C}_j^T\boldsymbol{e}_j \tag{10}$$

$$\boldsymbol{E}_L = \boldsymbol{e}_{Th,1} + \boldsymbol{e}_{Th,2} \tag{11}$$

$$\boldsymbol{Z}_{Th,j} = \boldsymbol{C}_j^T\boldsymbol{Y}_j^{-1}\boldsymbol{C}_j \tag{12}$$

$$\boldsymbol{Z}_T = \boldsymbol{Z}_{Th,1} + \boldsymbol{Z}_{Th,2} + \boldsymbol{R}_L \tag{13}$$

$\mathbf{e}_j$ is the vector of node voltages of subsystem j when solved independently of the rest of the subsystems, that is, with all the links open-circuited. $\mathbf{e}_{Th,j}$ and $\mathbf{Z}_{Th,j}$ are, respectively, the vector of Thévenin voltages and the matrix of Thévenin impedances of subsystem j as seen from the links. The set of equations for $\mathbf{I}_L$ in (8) states that the currents through the links can be calculated by substituting all subsystems with their respective Thévenin equivalents. After finding $\mathbf{I}_L$ the voltages in all subsystems can be found from the corresponding equations in (8).

At any given time, the standard MATE solution procedure for subsystems solved with the same integration step is as follows [23]:

1. Solve for the link currents using the Thévenin voltages of all subsystems calculated in the previous iteration.

$$\boldsymbol{I}_L = \boldsymbol{Z}_T^{-1}\boldsymbol{E}_L \tag{14}$$

2. Solve for the node voltages in all subsystems by injecting the link currents.

$$\boldsymbol{V}_j = \boldsymbol{e}_j - \boldsymbol{Y}_j^{-1}\boldsymbol{C}_j\boldsymbol{I}_L \tag{15}$$

3. For each subsystem evaluate the current injections **h**j that would be required for the next iteration at t=t+Δt.
4. Evaluate the Thévenin voltages of all subsystems for t=t+Δt.

$$\boldsymbol{e}_{Th,j} = \boldsymbol{C}_j^T\boldsymbol{Y}_j^{-1}\boldsymbol{h}_j \tag{16}$$

5. Increase the time by Δt and go back to step 1.

## IV. SFA-EMT Interfacing Protocol

### *A. Interfacing SFA and EMT simulators in MATE*

With the proper adaptations, interfacing SFA and EMT simulators using MATE can be accomplished similarly as in the EMTP multirate implementation in [23], since SFA uses an EMTP-like solution algorithm.

In the EMTP, the Thévenin voltage for the next solution time can be readily calculated from the solution at the present time step and, therefore, a Thévenin voltage for any instant in-between two consecutive solution times can be obtained by interpolation of the Thévenin voltages at those two solution times [25].

For two EMT subsystems (SS's), slow and fast, using a large and a small time step, respectively, the MATE multirate interfacing protocol is as follows [23]. At the beginning of the simulation, the two SS's are solved with the links open at their own first solution step and the Thévenin equivalents (TE's) for both SS's are obtained. Then, the solution of the fast SS for the next time steps begins. Using an interpolated value of the TE of the slow SS and the TE of the fast SS, the link currents are evaluated using the equations for the standard MATE solution. These currents are then injected into the fast SS to update it. A new TE of the fast SS is evaluated and time is advanced one small time step. The solution of the fast SS continues using the previous process, until it is time to update the slow SS. At this point, the fast SS has to provide its TE for the updating of the slow SS. Since the slow SS will be receiving a down-sampled version of the TE of the fast SS, in order to avoid aliasing errors this TE has to be low-pass filtered [23]. Using the TE of the slow SS and the low-pass filtered TE of the fast SS, the link currents are evaluated. These currents are then injected into the slow SS to update it. A new TE of the slow SS is evaluated and the solution of the fast SS begins again. The whole process is repeated until the maximum simulation time is reached. As opposed to previous multirate implementations (e.g., [24]), this procedure does not require the large time step to be an integer multiple of the small time step.

The SFA transformation allows the use of much larger times steps than would otherwise be required in a normal EMT simulation, but it turns the circuit variables into complex-valued variables. In a system partitioned into an SFA SS (slow SS) and an EMT SS (fast SS), intuitively, it would seem that the real-valued EMT solution should be interfaced with the real part of the SFA SS solution. However, this cannot be done directly, because the TE of the SFA SS is complex-valued, which requires that the combined MATE solution at the links level be carried out in terms of complex numbers. To keep the relationships in terms of complex variables, it is necessary to keep track of the response of the EMT SS in terms of both real and imaginary parts to combine with the SFA SS solution. It is a major contribution of the work presented in this paper that this idea can be achieved successfully. This concept has not been previously presented in the literature. This paper shows that this concept can be implemented efficiently and accurately by performing two simultaneous, parallel simulations, of the EMT SS, one for the real part and one for the imaginary part. The solution for the real part is the actual solution of the system, while the solution for the imaginary part maintains the analyticity of the corresponding complex-plane solution of SFA and allows the interfacing of the real EMT solution with the complex-plane SFA solution, taking full advantage of the benefits of the SFA transformation of using large time steps.

The SFA-EMT interfacing protocol is similar to the EMT-EMT MATE multirate protocol described previously, but with the updating of the SS's as indicated below.

### *B. Updating the EMT Subsystem*

Let us assume that the SFA SS (SS1) has already been updated at $t_1$ and is now time to update the EMT SS (SS2) at t, where $t_1 < t < t_1+\Delta T_{SFA}$. As indicated in Sub-section IV.A above, every time a SS gets updated, a TE of that SS is generated for the solution at the next time step. That is, after SS1 has been updated at time $t_1$, a vector of SFA equivalent Thévenin sources is calculated for the next solution at time $t_1+\Delta T_{SFA}$. By linearly interpolating between the two SFA equivalent Thévenin sources, a new TE of the SFA SS can be found to update SS2 at time t.

For SS1 at time t, and with the links connected, we have that

$$\bar{\boldsymbol{e}}_{Th1m}(t) = \bar{\boldsymbol{Z}}_{Th1}\bar{\boldsymbol{\iota}}_{Lm}(t) + \bar{\boldsymbol{v}}_{T1m}(t) \tag{17}$$

where $\bar{\boldsymbol{Z}}_{Th1}$ is the matrix of complex SFA Thévenin impedances of SS1 as seen from the links, and $\bar{\boldsymbol{e}}_{Th1m}$, $\bar{\boldsymbol{v}}_{T1m}$ and $\bar{\boldsymbol{\iota}}_{Lm}$, are the SFA vectors of the interpolated Thévenin sources and terminal voltages of SS1 and the currents through the links, respectively (direction of link currents is as in Fig. 1).

For updating the EMT SS, we need to solve for the link currents in normal time domain. Unshifting the SFA quantities in (17) by applying the inverse of the SFA transformation yields

$$\bar{\boldsymbol{e}}_{Th1}(t) = \bar{\boldsymbol{Z}}_{Th1}\bar{\boldsymbol{\iota}}_{L}(t) + \bar{\boldsymbol{v}}_{T1}(t) \tag{18}$$

where $\bar{\boldsymbol{e}}_{Th1}$, $\bar{\boldsymbol{v}}_{T1}$ and $\bar{\boldsymbol{\iota}}_{L}$, are the vectors of the interpolated Thévenin sources and terminal voltages of SS1 and the currents through the links, respectively, in normal time domain.

Let us consider both the real and imaginary solutions of the EMT SS at time t with the links connected. Let $\boldsymbol{e}_{Th2,R}(t)$, $\boldsymbol{e}_{Th2,I}(t)$, $\boldsymbol{v}_{T2,R}(t)$, $\boldsymbol{v}_{T2,I}(t)$, $\boldsymbol{i}_{L,R}(t)$ and $\boldsymbol{i}_{L,I}(t)$, be respectively, the vectors of real and imaginary Thévenin voltages, the voltages at the terminals of the links and the currents injected from the links into the EMT SS. Let the matrices of Thévenin resistances of the EMT SS, as seen from the links, be $\boldsymbol{R}_{Th2,R}$ and $\boldsymbol{R}_{Th2,I}$, for the real and imaginary solutions, respectively. Then for SS2, we have

$$\boldsymbol{v}_{T2,R}(t) = \boldsymbol{e}_{Th2,R}(t) + \boldsymbol{R}_{Th2,R}\boldsymbol{i}_{L,R}(t) \tag{19}$$

$$\boldsymbol{v}_{T2,I}(t) = \boldsymbol{e}_{Th2,I}(t) + \boldsymbol{R}_{Th2,I}\boldsymbol{i}_{L,I}(t) \tag{20}$$

Separating (18) into real and imaginary parts and combining with (19) and (20), considering the voltage drops in the link resistances, it follows that

$$(\boldsymbol{Z}_{Th1,R}+\boldsymbol{R}_l + \boldsymbol{R}_{Th2,R})\boldsymbol{i}_{L,R}(t) - \boldsymbol{Z}_{Th1,I}\boldsymbol{i}_{L,I}(t) = \boldsymbol{e}_R(t) \tag{21}$$

$$\boldsymbol{Z}_{Th1,I}\boldsymbol{i}_{L,R}(t) + (\boldsymbol{Z}_{Th1,R}+\boldsymbol{R}_l + \boldsymbol{R}_{Th2,I})\boldsymbol{i}_{L,I}(t) = \boldsymbol{e}_I(t) \tag{22}$$

where $\boldsymbol{R}_l$ is the matrix of link resistances and $\boldsymbol{e}_R(t)$ and $\boldsymbol{e}_I(t)$ are given by

$$\boldsymbol{e}_R(t) = \boldsymbol{e}_{Th1,R}(t) - \boldsymbol{e}_{Th2,R}(t) \tag{23}$$

$$\boldsymbol{e}_I(t) = \boldsymbol{e}_{Th1,I}(t) - \boldsymbol{e}_{Th2,I}(t) \tag{24}$$

Making

$$\boldsymbol{i}(t) = \begin{bmatrix} \boldsymbol{i}_{L,R}(t) \\ \boldsymbol{i}_{L,I}(t) \end{bmatrix} \tag{25}$$

$$\boldsymbol{e}(t) = \begin{bmatrix} \boldsymbol{e}_R(t) \\ \boldsymbol{e}_I(t) \end{bmatrix} \quad (26)$$

$$\boldsymbol{Z} = \begin{bmatrix} \boldsymbol{Z}_{Th1,R} + \boldsymbol{R}_l + \boldsymbol{R}_{Th2,R} & -\boldsymbol{Z}_{Th1,I} \\ \boldsymbol{Z}_{Th1,I} & \boldsymbol{Z}_{Th1,R} + \boldsymbol{R}_l + \boldsymbol{R}_{Th2,I} \end{bmatrix} \quad (27)$$

then, from (21)-(27)

$$\boldsymbol{Z}\boldsymbol{i}(t) = \boldsymbol{e}(t) \quad (28)$$

$$\boldsymbol{i}(t) = \boldsymbol{Z}^{-1}\boldsymbol{e}(t) \quad (29)$$

With (29), the real and imaginary link currents to update SS2 can be calculated. These currents are then injected into SS2 to update the voltages and currents within the EMT SS. After SS2 has been updated at t, the solution time is then advanced to t+$\Delta t_{EMT}$ and the process is repeated until it is time to update SS1.

Regarding the Thévenin equivalent of the SFA SS in (17), used for the determination of $\boldsymbol{i}(t)$ in (29) to update the EMT SS, it is worth noting that it has been derived from the discretization of the differential equations of the SFA SS and hence can capture the frequency response of the SFA subsystem for a range of frequencies around the fundamental, which is a function of the time step being used. For example, in a 60 Hz system, for a 4 ms time step in SFA (Nyquist frequency ($f_{Ny}$) = 125 Hz), frequencies of ±25 Hz in SFA (that is, $f_{Ny}/5$) can be accurately captured, which corresponds to a range in normal time domain from 35 to 85 Hz. This range can be increased by reducing the time step. This means that transients within the SFA SS or flowing into it through the boundaries, with frequencies within that range, can be accurately captured with the method, which makes it relevant in studies involving harmonic and sub-harmonic resonances and in stability studies with large frequency excursions.

### C. Updating the SFA Subsystem

Let us assume now that the EMT SS (SS2) has already been updated at $t_2$ and is now time to update the SFA SS (SS1) at t, where $t_2 < t < t_2+\Delta t_{EMT}$. As mentioned in Sub-section IV.A above, when updating the slow SS (SS1), a low-pass filtered or smoothed out version of the TE of the fast SS (SS2) must be used to evaluate the link currents to avoid aliasing errors in the solution. In this case, all the EMT Thévenin sources obtained in the EMT solutions (real and imaginary) after the last updating of SS1 are combined into a complex-number form, shifted with the SFA transformation and then averaged out as in [25, 26]. In this case, since the time steps need not be multiples, the averaging out is carried out by evaluating the integral below [26].

$$\bar{\boldsymbol{e}}_{Th2m,ave}(t) = \frac{1}{\Delta T_{SFA}} \int_{t-\Delta T_{SFA}}^{t} \bar{\boldsymbol{e}}_{Th2}(t) e^{-j\omega_0 t}\, dt \quad (30)$$

where $\bar{\boldsymbol{e}}_{Th2m,ave}(t)$ is the average value of the vector of shifted (SFA) complex EMT Thévenin sources over a period of time equal to $\Delta T_{SFA}$, and in this case it coincides with the fundamental frequency SFA phasors of the EMT Thévenin sources. This vector of fundamental frequency SFA phasors of the EMT Thévenin sources, but brought back to normal time domain, is the one to be used in the calculation of the link currents with the procedure indicated below.

For ease of calculation, when updating the SFA SS, it is also convenient to solve for the link currents in normal time domain and then shift the link currents back at the end of the calculation. By doing this, the evaluation of the link currents follows the same procedure as when updating the EMT SS and equations (17)-(29) equally apply, with the difference that in this case the SFA Thévenin sources are no longer interpolated sources but the actual SFA Thévenin sources calculated for the solution at time t and the EMT Thévenin sources are no longer the actual EMT Thévenin sources from the solution of the EMT SS at time t, but the unshifted SFA phasors that have been extracted from them using (30). After the real and imaginary link currents are evaluated with (29) then the SFA link currents in complex form are obtained with (31).

$$\bar{\boldsymbol{i}}_{Lm}(t) = (\boldsymbol{i}_{L,R}(t) + j\boldsymbol{i}_{L,I}(t))e^{-j\omega_0 t} \quad (31)$$

This vector of SFA link currents is then injected into SS1 to update the voltages and currents within this SS. After the SFA SS has been updated at t, the updating of SS2 begins again.

It is worth observing that, unlike all other hybrid techniques, with this approach, the fundamental frequency phasor is extracted from the EMT solution just by averaging the values in the SFA domain, without resorting to Fourier analysis or least-square fitting techniques.

### D. Selection of the boundary between SS's

Selecting the boundary between subsystems in a hybrid simulation is a compromise between computational efficiency and the accuracy of the solution. Ideally for maximum computational efficiency, the EMT subsystem should be as small as needed. The effect of this minimization will depend on the particular hybrid method used and the ratio between the large and small time steps. On the other hand, for maximum accuracy, the EMT subsystem should be made larger than just the region or the equipment of interest, so that the boundaries are sufficiently far from the source of harmonics and fast transients, but this would be detrimental to the computational efficiency.

Some hybrid TS-EMT techniques, (e.g., [6]), use an FDNE in the Thevenin equivalent used to represent the TS subsystem in the EMT subsystem, which makes it valid over a wide frequency range and allows the TS-EMT boundary to be moved closer to the source of transients without compromising the accuracy [2]. Thus, this type of techniques would require, in principle, a minimum buffer zone around the core part of interest in the EMT subsystem. Other TS-EMT methods (e.g., [4] and [7]), that use multi-port equivalents to represent the TS in the EMT subsystem and employ three sequence-based TS simulators, would require, in principle, a larger buffer region than the previous ones, since the impedances of the equivalents are evaluated at a single frequency. Hybrid TS-EMT methods, as in [8], that use simplified single-port Norton equivalents to represent the TS subsystem in the EMT subsystem and positive sequence-based TS simulators (e.g., [5] and [8]) would require a buffer region of a still bigger size to achieve satisfactory accuracy. On the other side, the extraction of the fundamental frequency variables at the boundary, which all hybrid methods need to do, gets easier and the accuracy improves, as the buffer region gets larger because the voltages and currents at the boundary get closer to sinusoidal and balanced. Thus, disposing of at least a minimal buffer region is in general desirable.

The proposed method uses multi-port Thévenin equivalents to represent all three phases of the SFA phasor subsystem as seen by the EMT subsystem and, as explained previously, these equivalents are valid for a frequency range around the fundamental frequency. These features of the proposed method ameliorate the problem of the buffer zone and good accuracy is obtained with boundaries relatively close to the source of transients and with large multirate ratios.

## V. Case Study

To assess the performance of the proposed interfacing protocol, the IEEE 39-bus test system [27] was simulated. This system was partitioned into two subsystems, a large one identified as SFA Subsystem (SS) or Subsystem 1 (SS1), and a small one, the EMT SS or SS2, as shown in the one-line diagram of Fig. 2. The boundary between subsystems was selected considering the closeness to the disturbance while maintaining the EMT SS small.

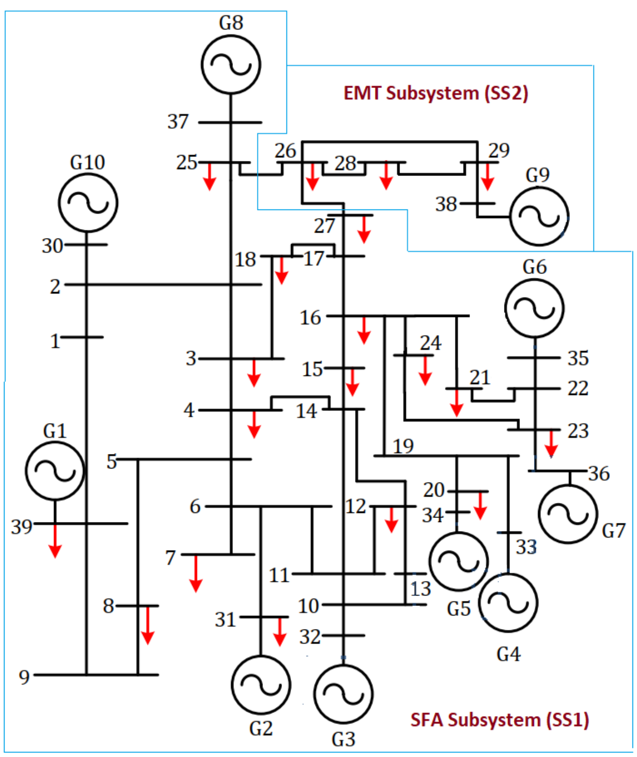


Fig. 2. IEEE 39-bus system [22] and SFA and EMT subsystems.

SS1 was simulated with the SFA solver, using a time step of 4 ms and SS2 was simulated with the EMT solver with a time step of 0.1 ms. The simulation consisted of the application of a 3-phase fault on the line between buses 26 and 28, very close to bus 28, at t = 50 ms, which was cleared after 8 cycles (60 Hz) by tripping that line. The dynamic behaviour of the system was recorded up to 3 s. All data was taken from [22] and the load in all buses was reduced by 20%. Generators were modelled with the classical model, lines with π-sections and loads were represented as constant impedances.

The simulator is coded in Python and consists of three solvers, the SFA and EMT solvers plus a link solver that executes the interfacing protocol. The SFA solver uses the backward Euler integration rule and the EMT solver uses the trapezoidal integration rule.

The purpose of the study is to evaluate the electromechanical transients in SS1 and to capture the electromagnetic transients in SS2 using the proposed interfacing protocol. To evaluate the performance of the protocol, the results are compared with a reference solution conducted by simulating the entire system with an EMT solver using a single small time-step of 0.1 ms.

In Figs. 3 and 4 the results for the SFA SS are presented. As can be seen there, the electromechanical transients in that SS are captured accurately with the proposed protocol. The error in frequency is less than 0.03%.

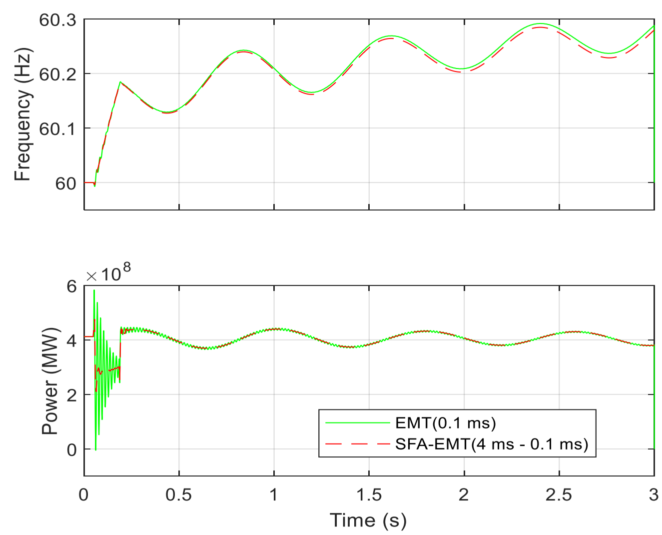


Fig. 3 Frequency and electrical power of generator 8 in SS1.

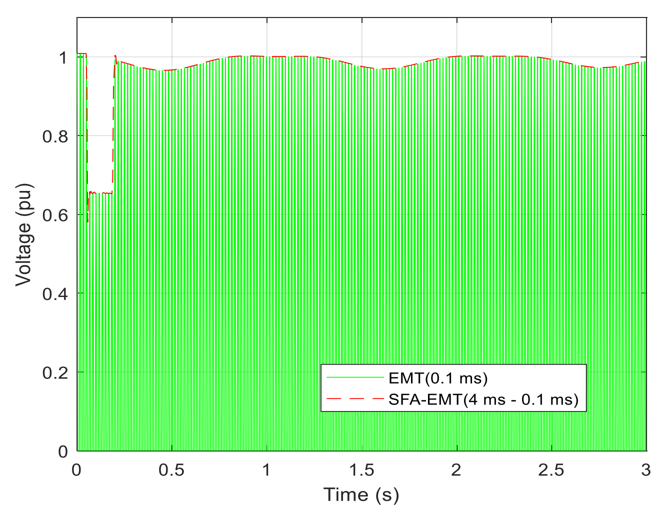


Fig. 4 Voltage of bus 27 (phase "a") in SS1.

In Fig. 4 it can be observed how the magnitude of the time-varying phasor of the voltage is nicely captured by the SFA solution of SS1.

In Figures 5 and 6 the results for the EMT SS are presented. Since we are interested in the fast-electromagnetic transients, only a window of about 200 ms is shown. From these results it can be observed that the hybrid simulator captures the electromagnetic transients in SS2 with good accuracy, the error in the peak current is 3.5 %.

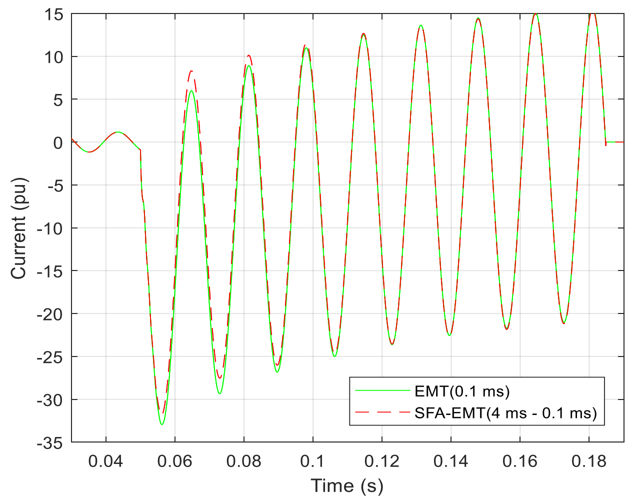


Fig. 5 Fault current from bus 28 (phase "c") in SS2.

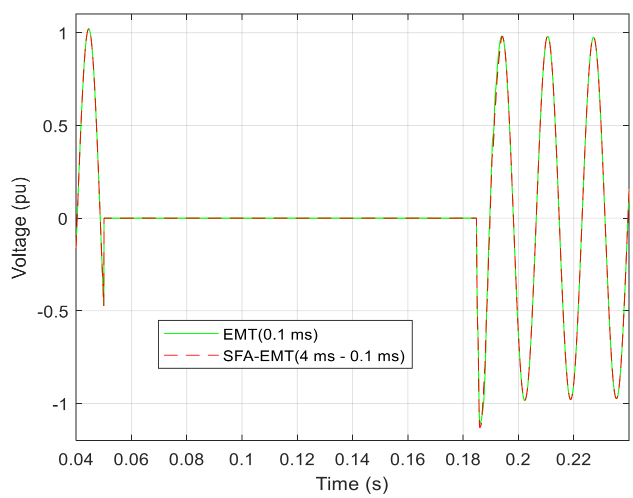


Fig. 6 Voltage at bus 28 (phase "c") in SS2.

## VI. Conclusions

This paper has presented the novel concept of using two parallel EMT simulations alongside an SFA simulation in a mixed complex-plane real-plane solution. This approach allows for direct simultaneous interfacing of hybrid phasor/EMT simulators within the MATE solution framework, without iterations and without the need for a time delay between subsystem solutions. The proposed approach maintains the mathematical property of analyticity of the phasor solution and synchronicity of the SFA/EMT hybrid simulator, while permitting the use of large time steps in the SFA solution. The method also allows the extraction of the fundamental frequency phasor of the EMT solution directly through the SFA transformation, without resorting to more elaborate techniques like Fourier analysis or least-squares fitting. This represents the first introduction of a new concept to interface phasor and EMT multirate systems in a combined closed-form solution.

The method has been successfully implemented in a new Python-based hybrid simulator that includes both SFA and EMT solvers, in addition to a third solver to execute the interfacing protocol. The validity of the proposed method was demonstrated using the IEEE 39-bus system for a case of transient stability. The results show that the proposed method exhibits very good accuracy for capturing the slow electromechanical transients in the SFA solution and, vice-versa, for capturing the fast electromagnetic transients in the EMT solution. The multirate nature of the solution provides considerable computational savings for large power systems when compared with a full-system EMT solution. The proposed framework allows for arbitrary time steps in each subsystem, and it is not restricted to these time steps being multiples. This provides generality in choosing the most adequate time step size according to the needs of each subsystem, thus further increasing the flexibility and efficiency of the simulation.

Future work will address issues related to energy exchange considerations due to aliasing and decimation when interfacing subsystems operating at different rates, as well as increasing the modeling capabilities of the hybrid simulator.

## VII. References


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