

Earth Return Admittance Impact on Crossbonded Underground Cables

Antonio P. C. Magalhães, Antonio C. S. Lima, Maria Teresa Correia de Barros

Abstract—Typically, the modeling of underground cables systems for electromagnetic transient simulations is carried out disregarding the effects of the ground return admittance. However, recent studies have presented precise models for the representation of the earth return admittance and demonstrated the importance of considering its effects on transient overvoltages along the cable and at sheaths crossbonding, especially if short minor sections are involved. In this paper, a study on the effects of the ground return admittance when considering an underground cross-bonded cable system is presented for 1 km and 300 m for the minor section length where the effects of earth return admittance are to be more pronounced. The inclusion of the ground-return admittance leads to a higher damping, which indicates that the conventional approach may lead to over conservative results.

Keywords: Electromagnetic transients, underground cables, ground return parameters.

I. INTRODUCTION

THE conventional approach for evaluating the transient responses of underground cables does not consider the inclusion of displacement current. For taking account this phenomenon, one needs to include the ground return admittance in the evaluation of the per unit length (pul) parameters. This can be carried out by using a modified expression for ground return impedance when compared with the traditional expressions proposed by Pollaczek [1] and the inclusion of ground return admittance. In the last two decades, the topic of wideband modelling of overhead transmission lines and underground cables considering ground displacement current has received some attention [2]-[19].

The evaluation of overvoltages in cable system when sheaths crossbonding is involved has been a topic that remained in the interest of the technical literature [12]-[18]. Recently, it was shown that intersheath modes circulating between grounding points [18]. Thus, this topology is suitable for the assessment of the impact of including the ground-return admittance in the transient voltage and current profiles

of a given cable system using crossbonding.

To obtain an accurate expression for the ground return impedance and admittance, one may need to consider distinct formulations. Reference [19] proposes expressions for the ground return impedance and admittance based on quasi-TEM (transverse electromagnetic) approximation of a full-wave model for a buried insulated cable [20]. One common challenge found to obtain ground return impedance and admittance is the numerical evaluation of the associated infinite integrals as this approach is the rather time-consuming and Gauss quadrature schemes are needed for the solution of the integrals [5][20]. To overcome this issue in [21] is proposed to use closed-form expressions for ground return admittance. In this reference it was shown that the proposed closed-form for the admittance together with the closed-form expression for ground return impedance proposed in [23] leads to a very accurate formulation with minimal mismatches for frequencies below a few MHz.

The results in [20] indicate that for shorter underground cables in high resistivity soils, the inclusion of the ground admittance plays a more significant effect. Thus, in this work we propose to investigate the impact of considering a ground return admittance in the transient response of a crossbonded cable configuration. The paper is organized as follows: the mathematical formulation of the pul parameter considering both the ground return impedance and admittance is presented in Section II. Section III shows the results for a given configuration of a crossbonded cable where a comparison between the conventional approach, i.e., neglecting ground displacement currents and the one proposed here is carried. To obtain the time responses a Numerical Laplace Transform (NLT) was applied [24]. The main conclusions are drawn in Section IV. The Appendices presents some details on deriving the pul parameters expression and a few results when a lower resistivity soil is considered.

II. MATHEMATICAL MODELING

A. Quasi-TEM approximation

Consider a system of insulated cables buried in a homogeneous soil with $\gamma = \sqrt{j\omega\mu_0(\sigma + j\omega\epsilon_r\epsilon_0)}$ where σ is its conductivity, ϵ_r is the relative permittivity, and μ_0 and ϵ_0 are the vacuum permeability and permittivity respectively. The mutual ground impedance between cables i and j is given by

$$Z_{g_{ij}} = \frac{j\omega\mu_0}{2\pi} (\Lambda_{ij} + S_{ij}) \quad (1)$$

and the mutual ground return admittance between the same cables is

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$$Y_{g_{ij}} = 2\pi(\sigma + j\omega\epsilon_r\epsilon_0)(\Lambda_{ij} - T_{ij}) \quad (2)$$

and where

$$\Lambda_{ij} = K_0(d_{ij}\gamma) - K_0(D_{ij}\gamma) \quad (3)$$

$$S_{ij} = \int_{-\infty}^{\infty} \frac{e^{-u_1(h_i+h_j)}}{u_1 + u_2} e^{-x_{ij}\lambda} d\lambda \quad (4)$$

$$T_{ij} = \int_{-\infty}^{\infty} \frac{u_1 e^{-\frac{u_1(h_i+h_j)}{2}} - e^{-u_1(h_i+h_j)}}{n^2 u_1 + u_2} e^{-x_{ij}\lambda} d\lambda \quad (5)$$

being K_0 the modified Bessel function of second kind and order zero, n is the reflection coefficient between ground and air, $u_1 = \sqrt{\lambda^2 + \gamma_s^2}$, $\gamma_0 = j\omega\mu_0\epsilon_0$, $u_2 = \sqrt{\lambda^2 + \gamma_0^2}$, d_{ij} is the distance between cables i and j and D_{ij} is the distance between the image of conductor i and conductor j .

The matrices are then assembled as

$$\mathbf{Z} = \mathbf{Z}_i + \mathbf{Z}_g \quad (6)$$

$$\mathbf{Y} = (\mathbf{Y}_i^{-1} + \mathbf{Y}_g^{-1})^{-1} \quad (7)$$

where \mathbf{Y}_i and \mathbf{Z}_i are the internal admittance and impedance matrices for the cable system used in the traditional approach where the ground displacement currents are neglected.

B. Closed-form approximation

For the closed-form approximation we consider the following:

$$Z_{g_{ij}} = \frac{j\omega\mu}{2\pi} \left(K_0(\gamma d_{ij}) + \frac{\ell^2 - x_{ij}^2}{D_{ij}} + K_2(\gamma d_{ij}) - 2 \frac{\ell^2 - x_{ij}^2}{\gamma^2 D_{ij}^4} (1 + \gamma\ell e^{-\gamma\ell}) \right) \quad (8)$$

$$Y_{g_{ij}} = 2\pi(\sigma + j\omega\epsilon_r\epsilon_0)(\Lambda_{ij} - \bar{T}_{ij}) \quad (9)$$

where Λ_{ij} is the same as before $\ell = h_i + h_j$ and \bar{T}_{ij} is

$$\bar{T}_{ij} = \frac{2\gamma^2}{\gamma_0^2 + \gamma^2} \log \left(\frac{2\gamma_0^2 + \gamma^2 \left(2 + \gamma_0 \sqrt{(h_i + h_j)^2 + x_{ij}^2} \right)}{\gamma_0^2 + \gamma^2 \left(1 + \gamma_0 \sqrt{(h_i + h_j)^2 + x_{ij}^2} \right)} \right) \quad (10)$$

The procedure to obtain the pul matrices is then identical to the one carried out considering infinite integrals.

III. TEST CASES

For the analysis of the transient response consider cable system configuration and cable data depicted in Fig. 1(a). In all tests, a relative ground permittivity of 10 was considered. The crossbonding configuration is presented in Fig. 2(a). **A step voltage is applied between sheathes, i.e., between nodes 4 and 6 as shown in Fig. 2. This connection scheme refers to inter-sheath mode excitation as presented in [19] and [21] and may be of interest for studies of incipient cable faults where a signal source is connected to cable sheath as presented in [28].** We have considered two possibilities for the lengths of the minor section, either 300 m or 1 km. Two values of the ground resistivity were considered, namely, $\rho = 500 \Omega m$ and $\rho = 1000 \Omega m$. In Appendix B, it is presented a few results that indicate that for soil resistivities below $200 \Omega m$, the ground return admittance play very little impact

The whole system was simulated using the Wolfram

Language considering Modified Nodal Analysis where the system matrix becomes a combination of the nodal admittance matrix and incident matrices and if needed some transfer functions. Therefore, the system matrix to be solved is better characterized as an immittance matrix. This can be understood as a generalization and a frequency domain counterpart of the MatEMTP [29]. As mentioned before, the NLT was used to obtain the time responses. To avoid aliasing a Hamming filter was used and 2048 frequency samples were considered. Figure 2(b) depicts the structure of the generalized immittance matrix used to solve the system.

In the following pictures, the label w/ Yext stands for the configuration where the ground-return admittance is considered, i.e., using (1) and (2) in the evaluation of ground-return admittance and impedance matrices. The conventional approach, i.e., neglecting ground displacement currents is presented with the label w/o Yext, which implies in considering $\mathbf{Y} = \mathbf{Y}_i$ a block diagonal matrix.

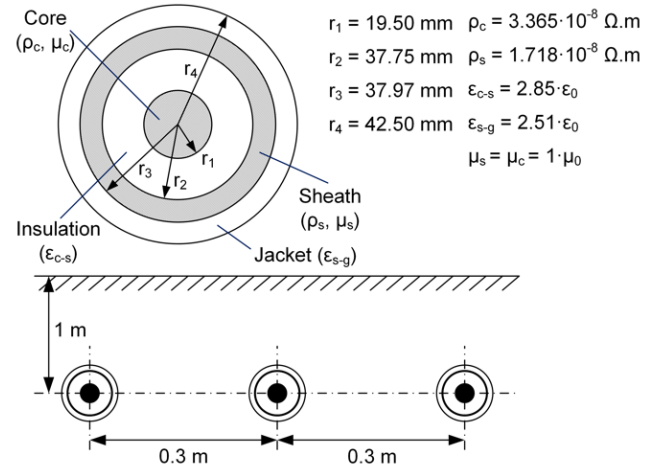
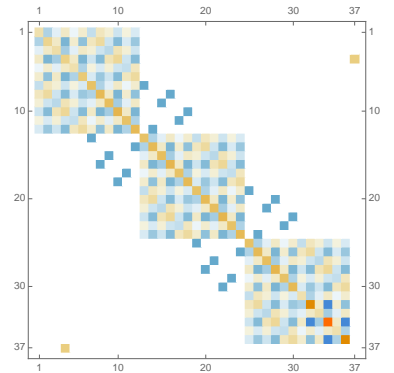
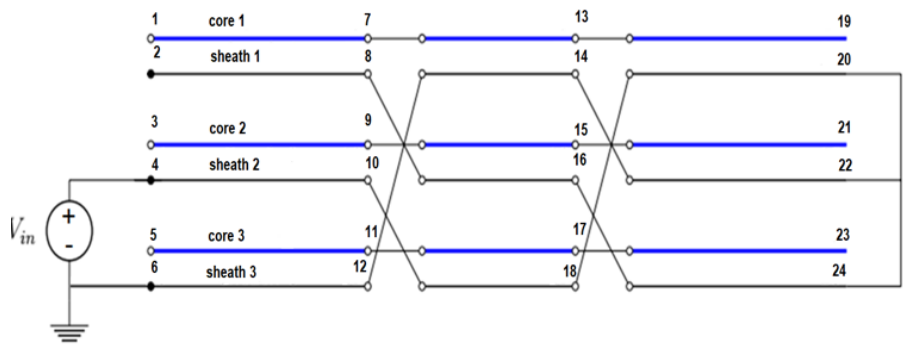


Fig. 1. Underground cable configuration and layout.

The impact of the inclusion of the ground-return admittance is evaluated considering initially a minor length of 1 km and then 300 m. Figures 3 and 4 depicts the voltages at node #7 (sending end of the first core) for both minor section lengths. Figures 5 and 6 depict the voltage at node #19 (sending end of the last core conductor) also considering both minor lengths. In is worth mentioning that there are some small but noticeable differences between the results obtained. These results indicate that even when core overvoltages are considered the presence of the ground-return admittance may slightly affect the results.

It is important to notice that for the conventional approach, we have considered the impact of the ground permittivity in the evaluation of the impedance as it is straightforward to do so. Conventional EMT modeling would not allow such formulation as it assumes the ground to be a good conductor, i.e., $\sigma \gg \omega\epsilon_r\epsilon$ instead of a dispersive medium.

Figures 5 and 6 present the behavior of the voltage at the same nodes but considering a minor length of 300 m. It can be observed that with shorter length, the mismatches are more pronounced even for a more conductivity soil.



(a) Cross-bonding configuration. (b) Generalized impedance matrix.
 Fig. 2. Circuit arrangement and structure of the generalized system matrix.

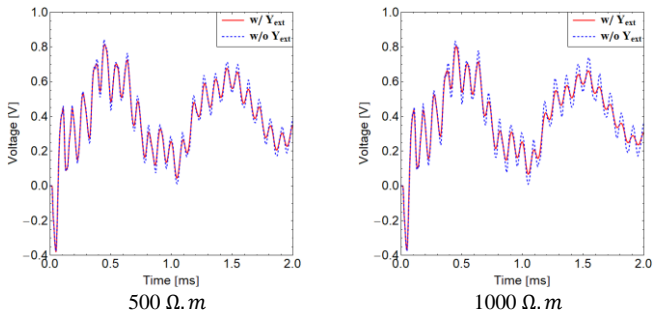


Fig. 3. Overvoltage at terminal #7 for a minor section of 1 km.

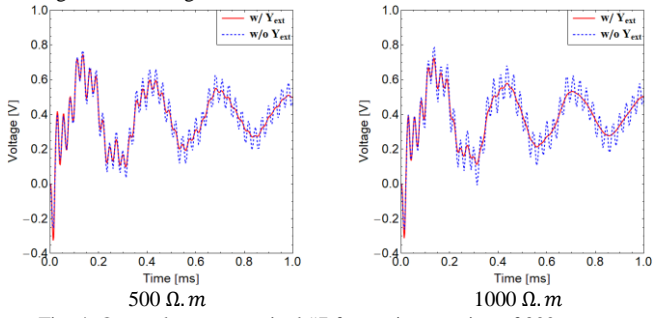


Fig. 4. Overvoltage at terminal #7 for a minor section of 300 m.

Figures 7 and 8 show the behavior of the voltage at node #12 (first sheath crossbonding “point”). It is worth mentioning that for this configuration if a minor section of 1 km is considered, the impact of the inclusion of ground displacement current is less pronounced than the one found in the core voltages.

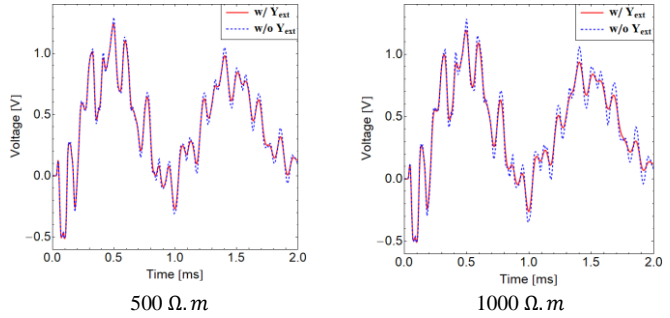


Fig. 5. Overvoltage at terminal #19 for a minor section of 1 km.

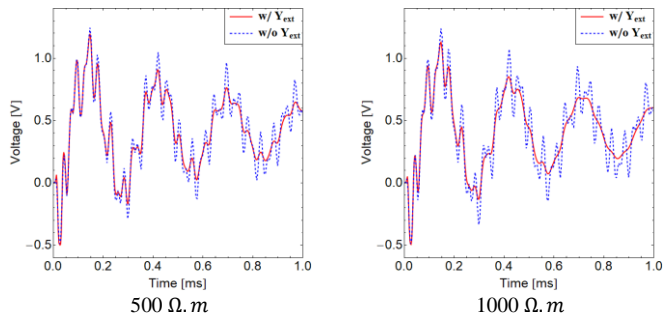


Fig. 6. Overvoltage at terminal #19 for a minor section of 300 m.

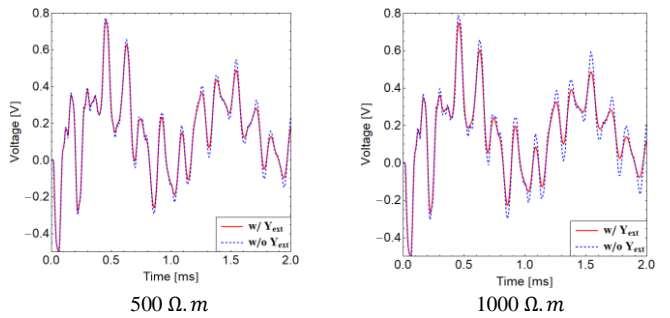


Fig. 7. Overvoltage at terminal #12 for a minor section of 1 km.

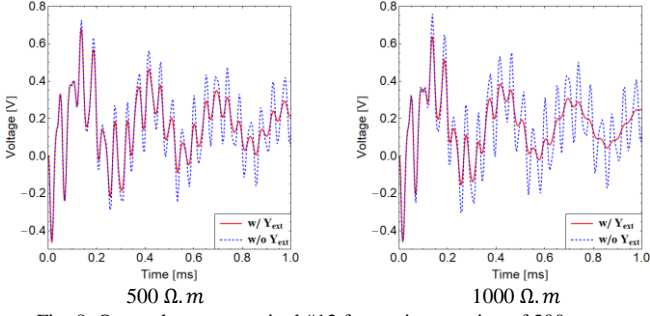


Fig. 8. Overvoltage at terminal #12 for a minor section of 300 m.

Figures 9 and 10 depict the overvoltage at the bonding of all sheaths (nodes #20, #22 and #24), i.e., it is the point where all sheaths are interconnect. This connection tends to minimize the effect of ground mode currents, as the sheaths are ungrounded. Thus, it can be noticed that only a very small mismatch appears.

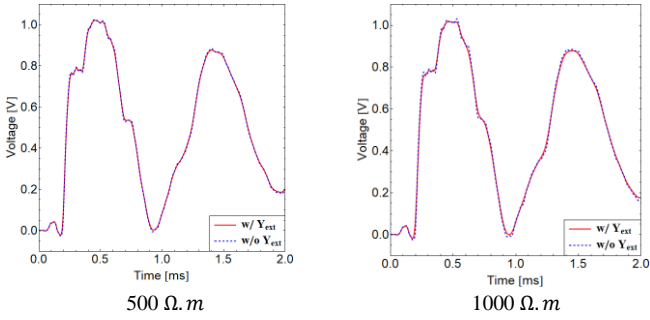


Fig. 9. Overvoltage at terminal #24 for a minor section of 1000 m.

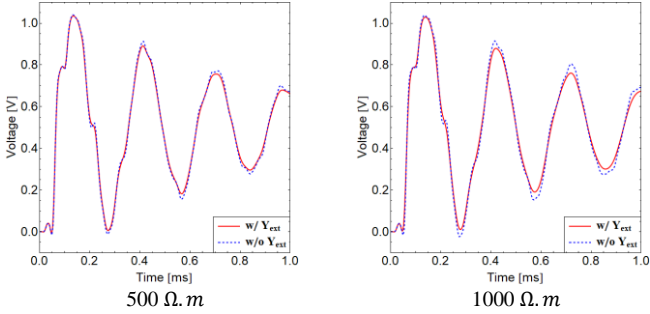


Fig. 10. Overvoltage at terminal #24 for a minor section of 300 m.

IV. CONCLUSIONS

This work has focused on the assessment of the impact of including the ground-return admittance in underground cable systems when crossbonding is considered. As in the conventional approach, it was found that the internal nodes are the ones subjected to the highest overvoltages. The inclusion of the ground-return admittance leads to a higher damping, which indicates that the conventional approach may lead to over conservative results.

The effect of the inclusion of ground-return admittance is more pronounced when shorter minor sections are considered and the soil has a higher resistivity, typically around $500 \Omega m$ or higher. Thus, configuration with short minor section in high resistivity soils should consider the effect of Y_g in the evaluation of pul parameters.

V. APPENDIX A DERIVATION OF PUL GROUND RETURN EXPRESSIONS

There are a number of possibilities to derive the integral equation to obtain that characterize a full-wave model for a infinite length conductor at a constant height in a two media system. We may consider using electric and magnetic vector potential, or the magnetic vector potential and the electric scalar potential or alternatively use Hertz vectors of magnetic and electric type. Regardless of the approach, the so-called modal equation reached is the same, see [30] for the details. Here, we consider the electric scalar potential and the magnetic vector potential. Thus, consider an infinitely long insulated conductor with outer radius r_e and at a constant height h parallel to the ground-air interface. The system of coordinates is such that the center of conductor is (x_c, y_c) . The injected current density vector is given by

$$\mathbf{J}_s = J_s \hat{\mathbf{z}} = \mathbf{I}_0 \exp(-\gamma_s z) \delta(x - x_c) \delta(y - y_c) \hat{\mathbf{z}} \quad (11)$$

where γ_s is the unknown propagation constant, \mathbf{I}_0 is an arbitrary complex constant. It is assumed that time-dependency is $\exp(j\omega t)$. Each medium i has a propagation constant given by $\gamma_i = \sqrt{j\omega\mu_0(\sigma_i + j\omega\epsilon_r\epsilon)}$. The electric and magnetic field in medium i can written as

$$\begin{aligned} (\nabla^2 + \eta_1^2)A_1 &= -\mu_0 J_s \\ (\nabla^2 + \eta_2^2)A_1 &= 0.0 \\ (\nabla^2 + \eta_1^2)\varphi_1 &= -\frac{\gamma_s}{\sigma_1 + j\omega\epsilon_r\epsilon} \mu_0 J_s \\ (\nabla^2 + \eta_2^2)\varphi_2 &= 0.0 \end{aligned} \quad (12)$$

where $\eta_i = \sqrt{\gamma_i^2 - \gamma_s^2}$, A_i , γ_i are related to medium i . A solution to (12) is possible using a double spatial Fourier Transform. There are two boundary conditions, the electric field at the conductor outermost coordinate, and the tangential components in E_1 and E_2 must be equal at the surface $(x, 0)$. After solving for the potentials one has to evaluate the electric field at the outermost layer of the conductor, i.e.,

$$E_{1z}(r, h) = -\frac{\mu_0 J_s}{2\pi} \left[\Lambda + S_1 - \frac{\gamma_s}{\gamma_1} (\lambda + S_2) \right] \quad (13)$$

Solving (13) leads to the determination of the propagation constant which in turn can be used to obtain all the other components of the electric and magnetic fields. Thus, the characteristic impedance Z_c is defined as the ratio between the injected current and the conductor voltage with respect to the ground. This voltage can then be expressed as

$$\mathbf{U} = \int_0^h E_{1y}(r, \xi) d\xi \quad (14)$$

then

$$\mathbf{Z}_c = (\varphi_{1h} - \varphi_{10}) + j\omega \int_0^h A_{1y}(r, \xi) d\xi \quad (15)$$

where φ_{1h} is the electric scalar potential evaluated at the outermost coordinate of the cable i.e., (r_e, h) . The ground-return pul parameters can be written as

$$\mathbf{Z} = \gamma_s \mathbf{Z}_c \quad (16)$$

$$Y = \gamma_s / Z_c \quad (17)$$

which then leads to the expression presented in Section II. Note that when the displacement current is disregarded the impedance and admittance formulations returns to the Pollaczek model. The displacement current is considered in the Maxwell's equations and the impedance and admittance are obtained from the determination of the boundary conditions of the electromagnetic fields at the boundaries between the outermost layers of the cable and the surrounding media and the boundary conditions of the electromagnetic fields at the air/soil interface. The procedure is described in [22] for the full-wave and quasi-TEM formulations of single conductor and in [19] for multi conductor systems.

VI. APPENDIX B SHEATH OVERVOLTAGE FOR LOW RESISTIVITY SOIL

In this appendix, it is shown the results for the sheath overvoltages for the same configuration, i.e., the one depicted in Fig. 2(a) but considering a soil resistivity of $100 \Omega m$ and the two possibilities for the minor section length, 300 m and 1 km respectively.

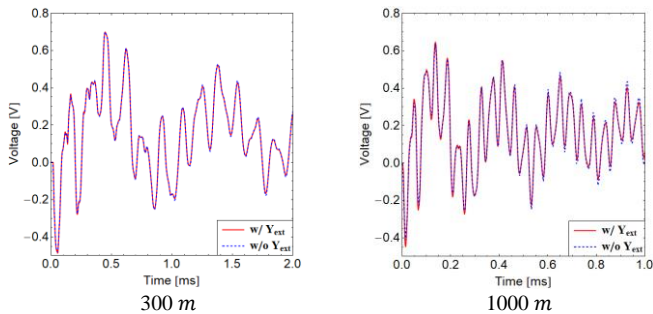


Fig. 11. Overvoltage at node #12 for a soil resistivity of $100 \Omega m$ considering a minor section of 300 m and 1 km.

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