# Application of a Performance Assessment Method to Identify the Applicability Range of Distribution Network Equivalent Models

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***Abstract*—The development of accurate equivalent models of distribution networks (DNs) is one of the most important aspects for power system dynamic analysis. Consequently, during the last decades, several equivalent models have been proposed to analyze the dynamic behavior of DNs. However, the performance of existing models is sensitive to several factors such as the pre-disturbance operating conditions and the penetration level of distributed generators. Scope of this paper is to evaluate the applicability range of conventional equivalent models for the dynamic analysis of modern DNs by using a recently proposed performance assessment method. Towards this objective, the performance, in terms of accuracy and generalization capability, of 22 conventional equivalent models is assessed. Finally, the most critical parameters of all examined equivalents are identified by applying a variance-based sensitivity analysis.**

***Index Terms*—Active distribution networks, dynamic equivalencing, measurement-based approach, power system dynamics, variance-based sensitivity analysis.**

## I. Introduction

Traditionally, power system dynamic analysis is conducted by using detailed network models [1]. However, the penetration of distributed generation (DG) units, the use of new types of loads, the need for more operational flexibility and the application of advanced voltage and frequency control strategies prevent system operators to develop and maintain accurate and up-to-date system models [2], [3]. Moreover, detailed information of the network structure, network assets and control parameters of DG units is rarely available to system operators. Thus, specific power system buses e.g., at key substations and feeders, are represented by equivalent models to simulate the aggregated behaviour of the downstream network, including lines, transformers, loads and DG units [3]. Equivalent models are also favored over detailed ones as they can overcome possible confidentiality issues that may come up when sharing information is needed regarding the distribution network (DN) operation [2]. For example, when coordination actions with transmission system operators are required.

Several equivalent models have been proposed in the literature. Early reviews were conducted in 1990s by the IEEE Task Force on Load Representation for Dynamic Performance [4]–[6] by summarizing the most known models and techniques. Later, with the advent of DGs and the smart grid technologies, in a response to the renewed interest in DN modelling, CIGRE Study Committee C4 established, in 2009, the Working Group (WG) C4.605: "Modelling and Aggregation of Loads in Flexible Power Networks". The aim of CIGRE C4.605 was to provide an updated overview of equivalent models to represent the aggregated dynamic behavior of modern DNs and step-by-step procedures for equivalent model development and validation [3]. In this context, the WG has conducted a survey campaign on industry practices regarding equivalent modelling. Results were published in [7], and the key findings of the questionnaire as well as the most important models being used were indicated. Recently, a thorough review on state-of-the-art equivalent modelling practices was presented in [8], reporting issues and new research trends on this topic.

Moving a step forward, in [9] a methodology to systematically evaluate the applicability of existing dynamic equivalent models for DN analysis has been proposed. In this regard, the accuracy and the generalization capability (robustness) of several equivalent models is evaluated and the accuracy of the examined equivalents is quantified using a set of key performance indicators. The generalization capability, i.e., the ability of the models to simulate disturbances different to those used for their development, is assessed by introducing a variance-based sensitivity analysis. Nevertheless, in [9] indicative results for only a specific DN configuration are presented. Hence, the generic nature of the proposed methodology and its constituent parts are not fully demonstrated. Additionally, the impact of DN topology on the accuracy and on the computational performance of the examined equivalents is not discussed.

This paper extends the work of [9] by: i) demonstrating the generic nature of the developed method. Towards this objective, simulations on the IEEE 33 Bus Test System are conducted and the performance of 22 equivalent models both in terms of accuracy and generalization capability is assessed. The comparative assessment of the results presented in this paper and in [9] reveal that the dynamic performance of DNs is considerably influenced by the grid topology, the relevant location of DG units and loads, and their mutual interaction. Therefore, evaluation techniques, as the proposed one, combined with modelling guidelines are required to facilitate and enhance dynamic analysis of DNs. ii) Thoroughly evaluating the proposed variance-based sensitivity analysis. For this

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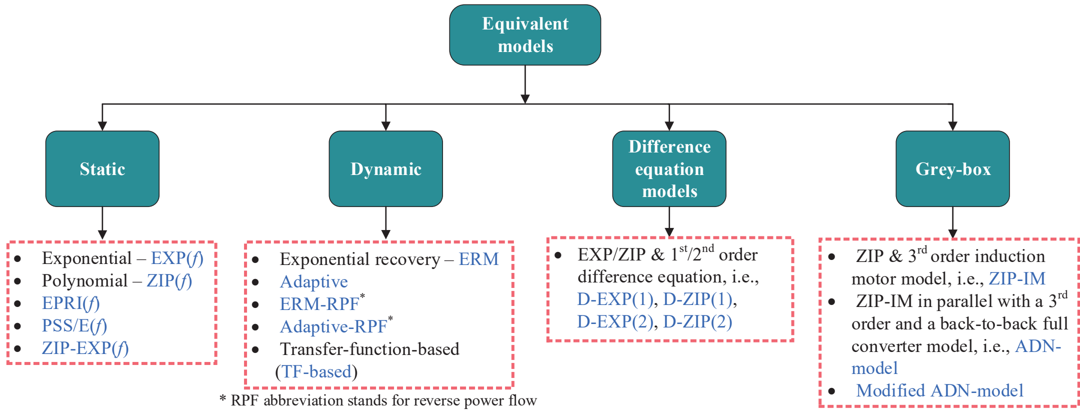


Fig. 1. Categories of equivalent models.

purpose, comparisons with the most-known generalization approach are performed. The conducted analysis provides further insights concerning the applicability and the advantages of the proposed method. iii) Identifying and ranking the most critical parameters of the 22 well-established equivalent models.

Following this introduction, the remaining of the paper is organized as follows. In Section II a brief description of the examined equivalent models is presented. In Section III, the simulation procedure is discussed. The accuracy of the examined equivalent models is quantified in Section IV. In Section V the generalization capability of the examined models is assessed and the most critical model parameters are identified. The ranking of critical parameters is also provided. Moreover, the efficacy of the proposed variance-based sensitivity analysis is evaluated against the conventional approach. An overview of the applicability range of the examined equivalents is provided in Section VI. Finally, Section VII summarizes the key findings and concludes the paper.

## II. Equivalent Models

Aggregated equivalent models can be grouped into static, dynamic, difference equation, and grey-box models [4], [6], [8], [9]. Using this classification, a total number of $N_{eq}$ = 22 equivalents has been identified and assessed in this paper. Classification results are summarized in Fig. 1. For each model, the most widely used abbreviated title (or the name of the software for which the model was developed, e.g., PSS/E) is adopted throughout the paper and it is denoted in the same figure with blue color. More details on the model type (static, dynamic, etc.), the mathematical formulation and the model parameters of the examined $N_{eq}$ = 22 equivalents are provided in [9]. Hereafter index $j$ $(j = 1, \ldots, N_{eq})$ denotes a specific model under consideration.

Static equivalents express the power at any time instant with respect to the voltage and/or frequency at that specific time instant [8]. These models are used for cases exhibiting near instantaneous time changes in power, following a voltage and/or frequency deviation. They can be also used if the interest is on the new steady-state rather than on the initial transient, e.g., long-term voltage stability studies. On the other hand, dynamic models express the power at any time instant as a function of voltage and/or frequency [8]. Dynamic models are preferred for dynamic studies, e.g., transient, frequency, and short-term voltage stability [4], [8].

Models based on difference equations constitute a more complex type of equivalents, consisting of static models and difference equations [10]. They result in very accurate estimates, but cannot provide insights concerning the physical properties of the examined system. Therefore, more sophisticated equivalent models have been introduced, namely grey-box, to represent more accurately the system under study [8]. Grey-box models can be used for the analysis of passive DNs, i.e., DNs that host only static and dynamic loads, as well as for the analysis of active DNs (ADNs), i.e., DNs that host both loads and DG units. In the former case, the so-called composite model is used [11], that consist of a static load and an induction motor (IM) model. In the latter case, more complex model structures are adopted including also components of inverter-interfaced DG units [12].

## III. Simulation Procedure

### A. System Under Study and Examined Scenarios

The examined system is shown in Fig. 2. It is a modified version of the IEEE 33 Bus Test System [13] with nominal grid voltage 12.66 kV. The total active and reactive demand is equal to 3.715 MW and 2.3 MVAr, respectively. DGs are modelled as full converter connected (FCC) units. Wind generators and PV units can be both represented by type 4 models in dynamic studies, since the converter can be considered to decouple the dynamics of the source on the DC part [14].

According to the guidelines of [9] a large number of dynamic responses is generated. The responses are created in terms of phasor simulations in DIgSILENT [15]. The DG penetration level is assumed varying from 0% to 100%, assuming a 20% step with respect to the total system load power; thus, a total number of six case studies $N_{DG}$ = 6 has been created. To achieve this, several DG units have been added per case study to the DN. Considering load, the rated power is constant assuming the proportion of the static and dynamic (IMs) part equal to 40% and 60%, respectively. Further details concerning the modelling of the system components (IMs, DG, static loads, etc.) are provided in [9].

For each one of the examined $N_{DG}$ case studies, five different operating conditions, i.e., discrete pre-disturbance

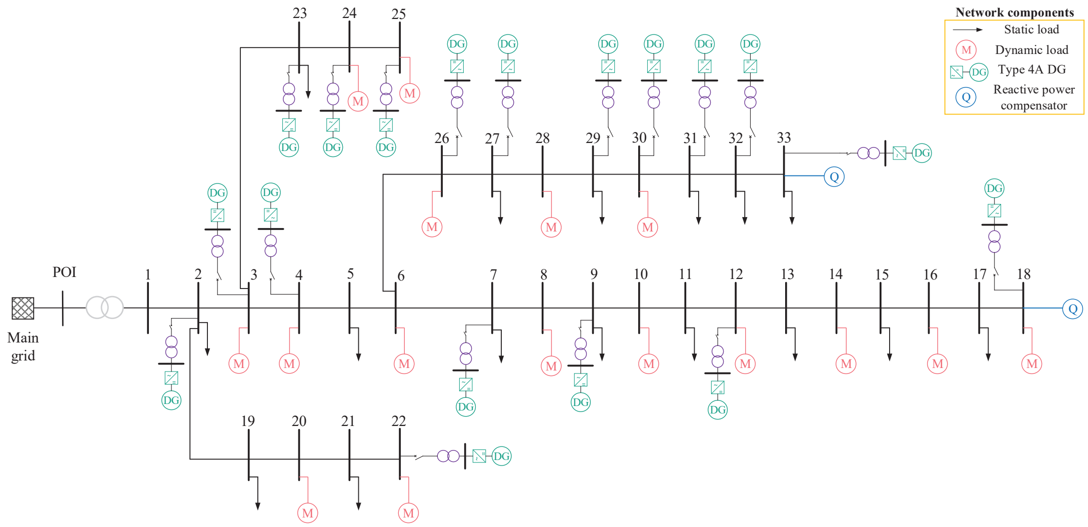


Fig. 2. Modified IEEE 33 bus distribution test system.

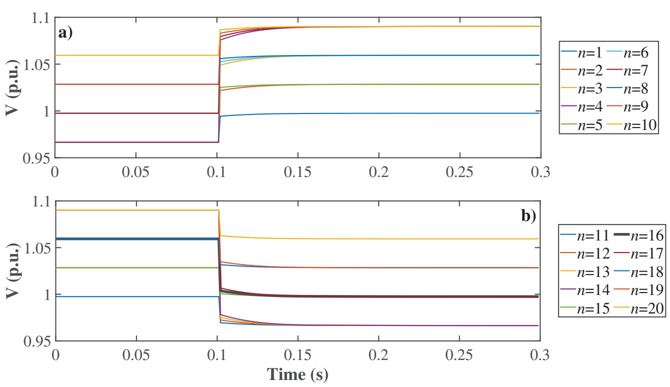


Fig. 3. Applied voltage disturbances. a) $n = 1, \dots, 10$; b) $n = 11, \dots, 20$.

voltage levels, are considered. The examined operating conditions are within the range of 0.97÷1.09 p.u. For each case study, $N_D = 20$ voltage disturbances, varying from -0.1 p.u. up to 0.1 p.u., are applied at the secondary side of the DN substation transformer (Bus 1 of Fig. 2) via tap-changing. All voltage disturbances are presented in Fig. 3. In summary, a total number of $N = N_{DG} \cdot N_D = 120$ scenarios is generated.

For each scenario, the voltage, frequency, active and reactive power dynamic responses at Bus 1 are acquired with a sampling rate equal to 1000 samples per second. The obtained responses are processed [16] and used to estimate equivalent model parameters. For this purpose, (1) is used,

$$J = \sum_{k=1}^{K} (y_n[k] - \hat{y}_n[k])^2 \qquad (1)$$

where, $K$ is the total number of samples, $y_n[k]$ is the real/reactive power response of the simulation model (as simulated in DIgSILENT) at the $k$-th sample of the $n$-th ($n = 1, \dots, N_D$) disturbance, and $\hat{y}_n[k]$ is the real/reactive power estimation provided using one of the $N_{eq}$ models.

### B. Applicability Assessment of Examined Models

The applicability of each examined model, $j \in \{1, ..., N_{eq}\}$, is evaluated per case study $t \in \{1, ..., N_{DG}\}$ in terms of accurate representation of the network dynamics and generalization capability (ability to represent network dynamics under new "unseen" disturbances, i.e., disturbances different to those originally used for the development of the model). Details on the two aspects of the evaluation procedure are provided in Sections IV and V, respectively.

## IV. Model Accuracy

### A. Adopted Metrics

The accuracy of the equivalent models is investigated for each case study, $t$, by using the indices of relative error ($\epsilon^t$), steady-state error ($SSE^t$), and overshoot error ($OE^t$), defined in (2), (3) and (4), respectively. Index $\epsilon^t$ is used to evaluate the accuracy of each model in terms of the overall response. $SSE^t$ and $OE^t$ are used to evaluate the accuracy of each equivalent regarding the modelling of the steady-state and the overshoot of the response, respectively. The assessment is performed in a statistical manner by calculating the median of the corresponding indexes for all $N_D$ dynamic responses contained at the $t$-th case. Real and reactive power responses are evaluated separately by using the three indexes.

$$\epsilon^t(\%) = \underset{n \in \{1,\dots,N_D\}}{\text{median}} \left( \frac{\sqrt{\sum_{k=1}^{K} (y_n[k] - \hat{y}_n[k])^2}}{\sqrt{\sum_{k=1}^{K} y_n^2[k]}} \cdot 100 \right) \qquad (2)$$

$$SSE^t(\%) = \underset{n \in \{1,\dots,N_D\}}{\text{median}} \left( \left| \frac{y_n^{ss} - \hat{y}_n^{ss}}{y_n^{ss}} \right| \cdot 100 \right) \qquad (3)$$

$$OE^t(\%) = \underset{n \in \{1,\dots,N_D\}}{\text{median}} \left( \left| \frac{y_n^{+} - \hat{y}_n^{+}}{y_n^{+}} \right| \cdot 100 \right) \qquad (4)$$

Here, $y^{ss}$ is the new steady-state power, $y^{+}$ is the power immediately after the disturbance; $\hat{y}^{ss}$ and $\hat{y}^{+}$ denote the corresponding estimates.

The resulting $\epsilon^t$, $SSE^t$ and $OE^t$ are evaluated over specific thresholds, i.e., $\tau_\epsilon$, $\tau_{SSE}$ and $\tau_{OE}$, respectively. If at least one of the predefined thresholds is violated, the equivalent model is considered inaccurate. Note that $\tau_\epsilon$, $\tau_{SSE}$ and $\tau_{OE}$ can be defined by interest entities, e.g., system operators and planners, according to their specific needs.

### *B. Model Accuracy Assessment*

The resulting $\epsilon^t$, $SSE^t$, and $OE^t$ for all examined models are depicted in Figs. 4, 5, and 6, respectively, by means of heat maps. Generally, all metrics increase with DG penetration. More details are provided in the next paragraphs.

As shown in Fig. 4a, concerning the modelling of real power, all equivalent models provide acceptable $\epsilon^t$ errors [17], i.e., error values lower than 5%, for DG penetration levels up to 60%. Nevertheless, for higher DG penetration levels, static models present noticeable performance degradation, leading to $\epsilon^t$ errors higher than 5%. Dynamic equivalents and grey-box models result into accurate estimates for DG penetration levels up to 80%. Based on the presented results it is clear that only the TF-based model and the equivalents based on difference equations (D-EXP(1), D-EXP(2), D-ZIP(1), and D-ZIP(2)) can provide accurate estimates for all examined case studies. Concerning the modelling of reactive power, as shown in Fig. 4b accurate estimates are obtained for all case studies by using the static models based on the ZIP formulation, i.e., ZIP, ZIP*f*, ZIP-EXP, ZIP-EXP*f*, as well as the EPRI*f*. However, it is worth noting that all other static equivalents fail to accurately simulate the reactive power behaviour, resulting in all cases to $\epsilon^t$ errors higher than 5%. Dynamic equivalents lead to accurate results up to 80% DG penetration level. The TF-based model, difference equation models and grey-box equivalents (apart fron ZIP-IM) can analyze accurately in all cases the dynamic behaviour of the reactive power.

Considering the new steady-state, results in Fig. 5a reveal that it can be efficiently analyzed by all models, since a $SSE^t$ lower than 5% is generally obtained. Similar accuracy is also observed for the reactive power modelling. Indeed, all models (apart from EXP, EXP*f*, and ZIP-IM) provide very accurate estimates. In particular, among the examined models, the most accurate estimates for the steady-state modelling of both real and reactive power by using the ZIP, the TF-based model, and equivalents based on difference equations.

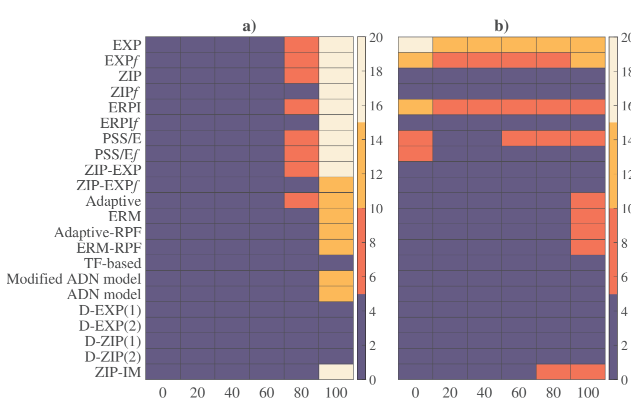


Fig. 4. $\epsilon^t$ for a) real and b) reactive power as a function of DG penetration.

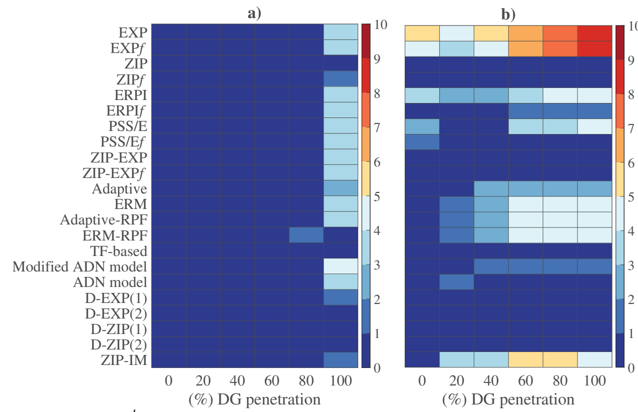


Fig. 5. $SSE^t$ for a) real and b) reactive power as a function of DG penetration.

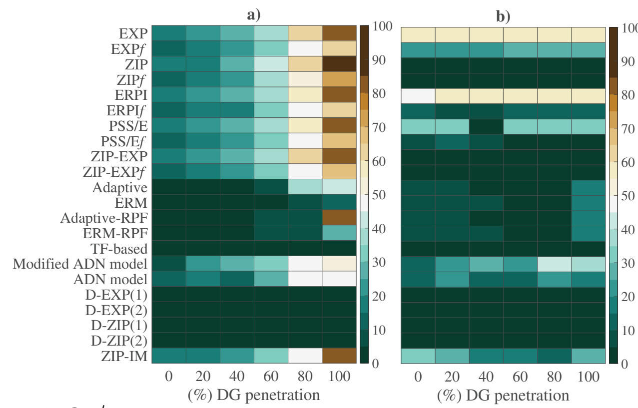


Fig. 6. $OE^t$ for a) real and b) reactive power as a function of DG penetration.

Results of Fig. 6a and 6b indicate that static models fail to capture real and reactive power overshoots. This is more marked for the modelling of real power, since $OE^t$ higher than 50% is reported in many cases. Dynamic models such as the adaptive, adaptive-RPF, ERM, and ERM-RPF provide accurate estimates for DG penetration levels up to 60%. Above this threshold, considerable performance degradation is observed. The investigation reveals that the most accurate models for the analysis of real and reactive power overshoots are the TF-based model as well as models based on difference equations.

## V. Generalization Capability

### *A. Variance-based Sensitivity Analysis*

The generalization capability of each model, $j$, is evaluated per case study in terms of global sensitivity analysis via (5):

$$\tau_{GCI,t} = \frac{1}{N_{eq}} \cdot \sum_{j=1}^{N_{eq}} \overline{GCI}_t^j \tag{5}$$

where $\tau_{GCI,t}$ is determined as the mean $\overline{GCI}_t^j$ of all models for the given case study. According to (6), $\overline{GCI}_t^j$,

$$\overline{GCI}_t^j = \frac{1}{N_{mp}^j} \cdot \sum_{i=1}^{N_{mp}^j} \frac{\sigma^j(\boldsymbol{E}_i)}{|\mu^j(\boldsymbol{E}_i)|} \tag{6}$$

is the standard deviation $\sigma^j(\boldsymbol{E}_i)$ of the $i$-th model parameter, $\theta_i$, with respect to its representative value, $\mu^j(\boldsymbol{E}_i)$. The calculations are based on the normal distribution assumption

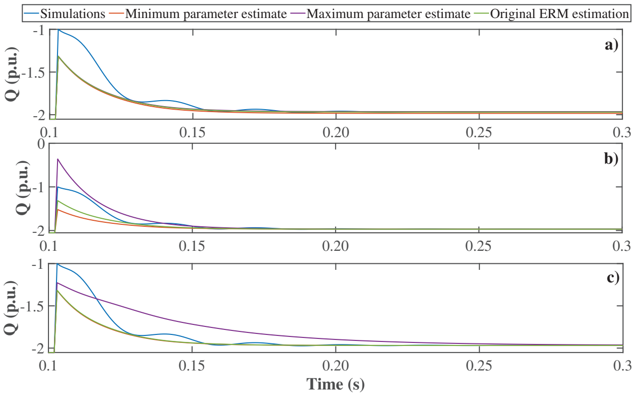


Fig. 7. Example for the proposed one-at-a-time sensitivity analysis using the ERM. Variation of a) $N_s$, b) $N_t$, and c) $T_y$ parameters. The mathematical formulation and the description of ERM parameters are given in [9].

[18]. $\boldsymbol{E}_i$ is a $N_D \times 1$ vector with the estimates of the $\boldsymbol{i}$-th real/reactive power model parameter. Note that, each model $j$ is represented with a set of real/reactive power parameters $\boldsymbol{\theta} = [\theta_1, \theta_2, \ldots, \theta_{N^j_{mp}}]$, where $N^j_{mp}$ denotes the total number of the real/reactive power model parameters, respectively. If $\overline{GCI}^j_t$ is less than $\tau_{GCI,t}$, a low dispersion in the parameter estimates is indicated corresponding to a robust set of model parameters. In this sense, the equivalent model is considered to present satisfactory generalization capabilities [9]. To ensure a fair comparison among the examined equivalent models, only the most critical parameters of each model, i.e., those parameters that have a significant impact on the model output, are used in the calculation of $\overline{GCI}^j_t$. These have been identified by means of the methodology presented in the next subsection.

### *B. Identification of Critical Model Parameters*

*1) One-At-a-Time Sensitivity analysis:* The most critical parameters of each model are identified by investigating the effect of the parameter variation on the model output per case study considering a set of reference responses, i.e., voltage, frequency, real, and reactive power. Specifically, the voltage reference response is defined as the voltage disturbance with level equal to the median of the $N_D$ disturbances. For example, among the $N_D = 20$ voltage disturbances presented in Fig. 3, the $16^{th}$ one is considered to be the reference voltage response. The frequency, real, and reactive power reference responses per case study are those corresponding to the voltage reference disturbance. Using the reference responses the model parameters, $\boldsymbol{\theta}^{ref}$, are identified.

Subsequently, each estimated $i$-th model parameter $\theta_i^{ref}$ ($\theta_i^{ref} \in \boldsymbol{E}_i$) is varied one-at-a-time (the remaining parameters are constant) within specific limits and $\epsilon_n$ is computed. These limits are defined by the minimum and maximum parameter estimates of the $N_D - 1$ responses, excluding the reference response. Metric $\epsilon_n$ is used to quantify the mismatch between the reference response and the response obtained by applying the varied set of parameters. In this context, $y_n[k]$ and $\hat{y}_n[k]$ in (2) refer to the responses obtained with the reference and the varied parameters, respectively. High $\epsilon_n$ values indicate a significant influence of the target model parameter. Therefore, a model parameter is considered critical, if at least one of the two aforementioned parameter variations lead to $\epsilon_n$ higher than $\tau_\epsilon$ for both the real and the reactive power.

*2) Indicative Example:* To elucidate on the proposed sensitivity analysis an indicative example is presented in Fig. 7. In this figure the blue line denotes the reference reactive power response for the 0% DG penetration case study. The reference response is obtained using the detailed DN model in DIgSILENT. The green response denotes the estimate provided by the ERM using the reference parameters, i.e., ERM parameters estimated using the voltage reference response and the depicted reference reactive power response. The orange and purple responses denote the ERM estimates derived using the minimum and maximum parameters (calculated in terms of the the remaining $N_D - 1$ responses), respectively.

As shown in Fig. 7a the impact of parameter $N_s$ on the overall reactive power response is trivial. Indeed, $\epsilon_n$ for the reference response is 4.6, while for maximum and minimum parameter variation is 4.7 and 4.9, respectively. On the other hand, the variation of $N_t$ and $T_y$ parameter results in $\epsilon_n$ values equal to 6.5 and 5.4, respectively, revealing the significant influence of these parameters on the overall model accuracy.

*3) Critical Model Parameters:* The model critical parameters are identified by using the proposed one-at-a-time sensitivity analysis and are reported in Table I. For the analysis,

TABLE I
CRITICAL MODEL PARAMETERS IDENTIFIED IN MOST CASE STUDIES.

| Model structure | Real power | Reactive power |
|---|---|---|
| EXP | $K_{yV}$ | $K_{yV}$ |
| EXP$f$ | $K_{yV}$ | $K_{yV}$ |
| ZIP | $p_1, p_2$ | $p_1, p_2$ |
| ZIP$f$ | $p_1, p_2$ | $p_1, p_2$ |
| EPRI | $K_{pv1}, K_{pv2}, P_{a1}$ | $K_{qv2}, K_{qv1}, Q_{a1}$ |
| EPRI$f$ | $P_{a1}, K_{pv1}, K_{pv2}$ | $K_{qv2}, K_{qf2}, K_{qf1}$<br>$K_{qv1}, Q_{a1}$ |
| PSS/E | $a_1, n_2, n_3$ | $n_3, a_1, a_2, n_1, n_2$ |
| PSS/E$f$ | $n_1, n_2, a_1, n_3, a_2$ | $n_1, n_3, a_2, a_1, n_2$ |
| ZIP-EXP | $n_{yv2}, K_{y1}, K_{y2}$,<br>$K_{yc}, n_{yv1}, K_{yi}$ | $K_{y2}, n_{yv2}, n_{yv1}$,<br>$K_{y1}, K_{yc}, K_{yi}$ |
| ZIP-EXP$f$ | $n_{yv1}, K_{y2}, K_{yf1}, K_{yc}$<br>$K_{yi}, K_{y1}, K_{yf2}, n_{yv2}$ | $K_{y1}, K_{y2}, n_{yv2}, K_{yc}$<br>$K_{yi}, K_{yf1}, K_{yf2}, n_{yv1}$ |
| Adaptive | $N_s, T_y$ | $T_y, N_t$ |
| ERM | $N_t, T_y$ | $N_t, T_y$ |
| Adaptive-RPF | $T_y, \alpha_1$ | $T_y$ |
| ERM-RPF | $\alpha_1, \beta_1$ | $T_y, \beta_1$ |
| TF-based | $\kappa_1, \kappa_2$ | $\lambda_1, \lambda_2$ |
| D-EXP(1) | $a, c_{y0}, a_{y1}, c_{y1}, K_{yV}$ | $K_{yV}, c_{y0}, c_{y1}, a_{y1}, a$ |
| D-EXP(2) | $a, c_{y1}, a_{y1}, c_{y2}$,<br>$K_{yV}, a_{y2}, c_{y0}$ | $K_{yV}, c_{y1}, c_{y0}, a_{y1}$,<br>$c_{y2}, a_{y2}, a$ |
| D-ZIP(1) | $a, p_1, p_2$,<br>$c_{y0}, c_{y1}, a_{y1}$ | $p_1, c_{y0}, p_2$,<br>$a, c_{y1}, a_{y1}$ |
| D-ZIP(2) | $c_{y0}, p_1, a, p_2$,<br>$c_{y1}, a_{y1}, c_{y2}, a_{y2}$ | $p_1, p_2, c_{y1}, c_{y0}$,<br>$c_{y2}, a, a_{y1}, a_{y2}$ |
| ZIP-IM | $P_Z, P_I, X'_m$<br>$\delta_m, X_m, T'_{dm}$ | $Q_Z, Q_I, X'_m$<br>$\delta_m, X_m, T'_{dm}$ |
| Modified ADN model | $P_Z, P_I, X_m, T'_{dm}$,<br>$T'_{dg}, \delta_g, E_{FD}, \delta_m$,<br>$X'_m, X_g, X'_g$ | $Q_Z, Q_I, T'_{dg}, \delta_g$,<br>$E_{FD}, \delta_m, X_m, T'_{dm}$,<br>$X'_m, X'_g, X_g$ |
| ADN model | $P_Z, P_I, T'_{dg}, \delta_g$,<br>$E_{FD}, \delta_m, X_m, T'_{dm}$,<br>$X'_m, X'_g, X_g$ | $Q_Z, Q_I, E_{FD}, \delta_g$,<br>$T'_{dg}, \delta_m, X_m, T'_{dm}$,<br>$X'_m, X'_g, X_g$ |

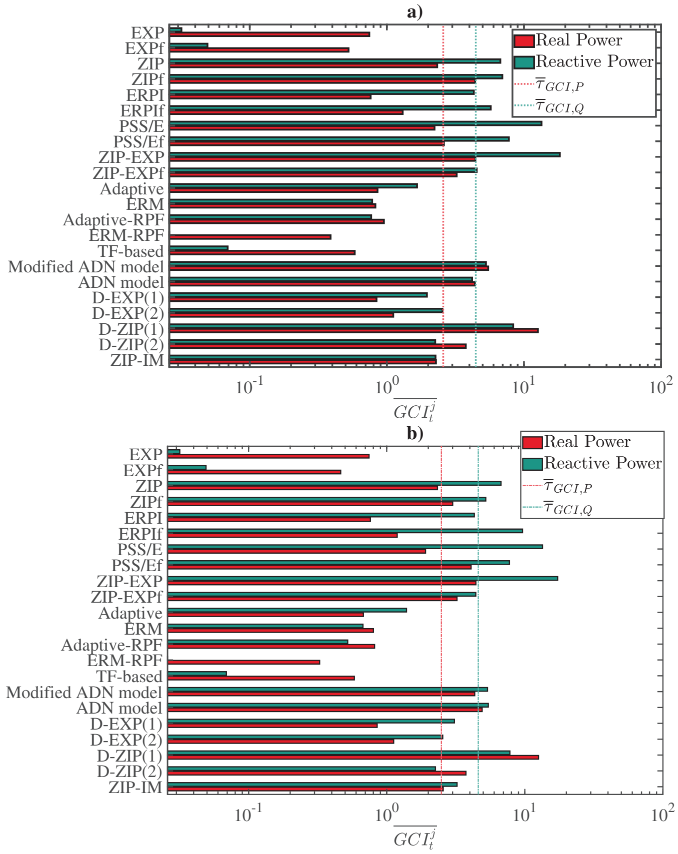


Fig. 8. Mean real and reactive power $\overline{GCI}_t^j$. a) All model parameters, and b) only critical ones, are considered during the calculation of $\overline{GCI}_t^j$.

$\tau_\epsilon = 5\%$. Note that the critical model parameters are arranged in a ranked order, with the most important appearing first and the rest following in a descending order.

### C. Quantification of Generalization Capability

In Fig. 8a, the resulting mean $\overline{GCI}_t^j$ is presented, as computed for the critical model parameters identified for each equivalent. Note that for each of the examined equivalents the mean value of $\overline{GCI}_t^j$ across the considered cases is summarized by means of bars. The mean values of $\tau_{GCI,t}$ for real and reactive power modelling, i.e., $\overline{\tau}_{GCI,P}$ and $\overline{\tau}_{GCI,Q}$, are also plotted as vertical dashed lines. Note that $\overline{\tau}_{GCI,P}$ and $\overline{\tau}_{GCI,Q}$ are determined as the mean $\tau_{GCI,t}$ values computed across the $N_{DG}$ cases.

Results indicate that the examined dynamic models result in relatively low $\overline{GCI}_t^j$ values, thus present high generalization capabilities. On the other hand, static models (apart from EXP and EXP*f*), grey-box equivalents as well as models based on difference equations exhibit lower generalization capabilities since they generally result in higher $\overline{GCI}_t^j$ values.

$\overline{GCI}_t^j$ index varies according to the number of the selected model parameters used for its calculation. In Fig. 8b, $\overline{GCI}_t^j$ has been also computed taking into account all model parameters for each equivalent. By juxtaposing Fig. 8a and Fig. 8b, it is evident that the $\overline{GCI}_t^j$ value does not vary noticeably. However, as justified in Section V-D the calculation of the $\overline{GCI}_t^j$ index using only the critical model parameters results into more accurate remarks considering the generalization capability of the equivalent models.

### D. Comparison with other generalization approaches

The generalization capability of equivalent models is generally assessed in the literature by dividing the available data into training and validation sets [2], [11]. Generic model parameters are obtained using the training data set, whereas the validation set is used to validate the efficiency of the derived generic models. Nevertheless, the development of a generic model that fits well to new unseen disturbances depends on the partitioning of the dataset into training and validation sets as well as on the availability of plethora of dynamic responses.

Therefore, the variance-based sensitivity analysis metric approach has been adopted. By this means the model sensitivity is directly interpreted; thus, a physical insight on model dynamics is provided. The most important advantage of this approach is that it can be applied even to cases with limited data (as it is the usual case regarding measurement availability for DNs).

The proposed index $\overline{GCI}_t^j$ determines the extent of variability of model parameters in relation to the corresponding mean values. In this way, the sensitivity of the model output with respect to the parameter variation is quantified. Specifically, high $\overline{GCI}_t^j$ values indicate significant dispersion of model parameters that consequently reveals difficulty to predict new operating conditions. On the contrary, if $\overline{GCI}_t^j$ is low, the model estimates do not vary significantly. This is an indication that the derived model parameters are more robust and thus can apply to a wider range of operating conditions.

Therefore, by using the proposed $\overline{GCI}_t^j$ index an insight regarding the generalization capabilities of a model structure is provided. It shall be noted that the proposed $\overline{GCI}_t^j$ index is a standardized measure of dispersion; thus, it can be used to assess the variability of model estimates with widely different mean values for different models. A fair comparison among the models is ensured, since $\overline{GCI}_t^j$ is computed for each model on the basis of the most critical parameters. Most importantly, the evaluation of the generalization capabilities of each model per case study is not carried out in terms of a fixed value, but on $\tau_{GCI,t}$, which is defined as the mean $\overline{GCI}_t^j$ of all models, being a representative metric per case study and representative indicator of the generalization capabilities of the models.

To validate that the proposed variance-based index can provide reliable information regarding the generalization capability of equivalent models, the following analysis is performed. The 20% DG penetration case study is considered and representative/generic parameters are computed for the D-ZIP(1) and the ERM as the median of the corresponding parameters obtained by all the $N_D$ disturbances. This approach is similar to the one presented in [11]. The D-ZIP(1) and the ERM are indicatively selected for the analysis, because as shown in Figs. 4-6 they present practically the same accuracy for the real power modelling. Afterwards, the generic representations of D-ZIP(1) and ERM, i.e., instances of the D-ZIP(1) and the

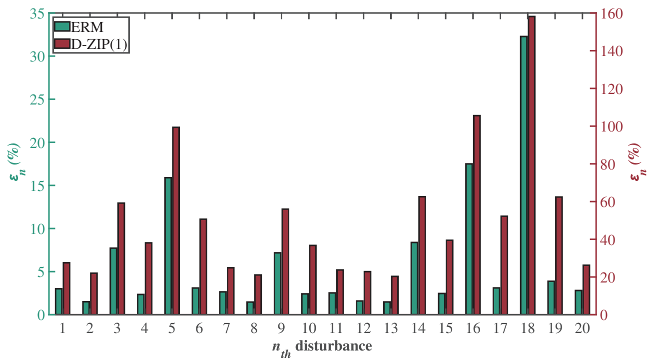


Fig. 9. $\epsilon_n$ for real power modelling using representative/generic parameters.

ERM that employ the generic parameters, are used to simulate real power responses of the original $N_D$ disturbances. Subsequently, the relative error $\epsilon_n$ ($n = 1, \ldots, N_D$) is computed assuming the original real power responses and those derived by the generic representations of the D-ZIP(1) and the ERM. The $\epsilon_n$ for all $N_D$ dynamic responses is depicted in Fig. 9.

From Fig. 9 it is clear that equivalents based on the ERM structure are more robust compared to those derived using the D-ZIP(1) structure. Indeed, the median $\epsilon_n$ for the ERM is 2.91%, while for the D-ZIP(1) is 38.79%. Additionally, in all cases ERM results in lower $\epsilon_n$ values compared to the D-ZIP(1). Thus, it can be perceived that the ERM presents higher generalization capabilities compared to the D-ZIP(1) model.

The same remark can also be deduced by using the proposed variance-based index. Indeed, for the examined case, $\tau_{GCI,t}$ is 2.47; $\overline{GCI}_t^j$ for the ERM is merely 0.52, indicating that this model presents satisfactory generalization capability. On the contrary, $\overline{GCI}_t^j$ for the D-ZIP(1) is 14.33. This significant $\overline{GCI}_t^j$ value denotes that the D-ZIP(1) model exhibits low generalization capability.

To demonstrate the qualitative effect of calculating the $\overline{GCI}_t^j$ index over only the critical parameters of each model, the following analysis is conducted. Let us consider reactive power modelling using indicatively the ZIP$f$ model for the 0% penetration case, and examine two different scenarios. In the first, all model parameters are taken into consideration during the calculation of the index, whereas in the second only the most important ones. $\overline{GCI}_t^j$ is computed equal to 4.02 and 6.99 in the former and the latter scenario, respectively; the corresponding $\tau_{GCI,t}$ is 5.11 and 5.14. Therefore, for the first scenario the ZIP$f$ model is deemed to present adequate generalization capability, whereas for the second inadequate. To validate the outcomes that stem from the two scenarios, a generic ZIP$f$ equivalent model is developed (following the procedure described earlier), and compared to the original reactive power responses. The calculated median $\epsilon_n$ from the $N_D$ responses is 18.98% $> \tau_\epsilon$, indicating that the developed generic model is not robust enough as substantiated by the application of the proposed approach, i.e., second scenario.

Based on the above, it can be deduced that by calculating the proposed variance-based index considering only the critical model parameters more reliable information concerning the generalization capability of equivalent models can be provided.

## VI. Overall Assessment of the Examined Models

In this Section, the overall performance and computational efficiency of the examined equivalents is investigated.

### A. Analysis of Overall Performance & Computational Burden

The overall performance of the examined equivalents in terms of generalization capability and accuracy is investigated per case study, i.e, per DG penetration level. Models that result simultaneously in real and reactive power $\epsilon^t$, $SSE^t$, and $OE^t$ lower than 5% are considered as accurate [9]. Additionally, models that exhibit $\overline{GCI}_t^j < \tau_{GCI,t}$ are considered as robust, i.e., that they present satisfactory generalization capability.

Table II provides a performance summary of all examined equivalents taking into account both their accuracy and generalization capabilities. A red color denotes inaccurate models, i.e., at least one of the adopted error metrics is higher than 5%. A yellow color denotes an accurate equivalent exhibiting low generalization capabilities ($\overline{GCI}_t^j > \tau_{GCI,t}$). A green color stands for an accurate equivalent that also possesses satisfactory generalization capability, i.e., $\overline{GCI}_t^j < \tau_{GCI,t}$.

Results reveal that the examined static and grey-box models cannot accurately simulate the dynamic behavior of the examined DN. At this point it must be noted that ADN grey-box models fail to accurately analyze system dynamics due to their structure which *a priori* assumes that small synchronous generators are installed in the DN [12]. Among the examined dynamic equivalents the TF-based model presents high accuracy and also high generalization capabilities under all examined cases. The rest of the examined dynamic models provide accurate estimates only under low DG penetration levels. Accurate estimates are also obtained by difference

TABLE II
Performance summary of the examined equivalent models for the dynamic analysis of the IEEE 33 Bus Test System.

| **Model structure** | **DG penetration (%)** | | | | | |
|---|---|---|---|---|---|---|
| | **0** | **20** | **40** | **60** | **80** | **100** |
| EXP | | | | | | |
| EXP$f$ | | | | | | |
| ZIP | | | | | | |
| ZIP$f$ | | | | | | |
| EPRI | | | | | | |
| EPRI$f$ | | | | | | |
| PSS/E | | | | | | |
| PSS/E$f$ | | | | | | |
| ZIP-EXP | | | | | | |
| ZIP-EXP$f$ | | | | | | |
| Adaptive | | | | | | |
| ERM | | | | | | |
| Adaptive-RPF | | | | | | |
| ERM-RPF | | | | | | |
| TF-based | | | | | | |
| D-EXP(1) | | | | | | |
| D-EXP(2) | | | | | | |
| D-ZIP(1) | | | | | | |
| D-ZIP(2) | | | | | | |
| ZIP-IM | | | | | | |
| Modified ADN model | | | | | | |
| ADN model | | | | | | |

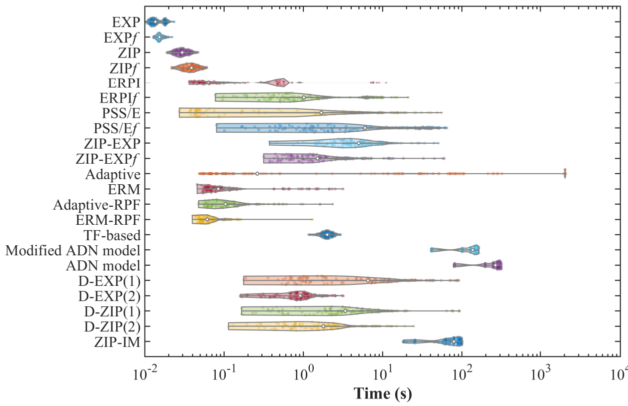


Fig. 10. Required execution time for real and reactive power modelling.

equation based models. Nevertheless, only D-EXP(1) and D-EXP(2) possess satisfactory generalization capabilities.

The resulting execution time for the $N$ cases is statistically analysed in Fig. 10 by means of violin plots for each model. For the model calculations an i7-8550U, 1.8 GHz, RAM 8 GB personal computer was used.

### B. Discussion

Here, the results of Table II and Fig. 10 are compared with the corresponding results presented in [9] (results of Table II and Fig. 10 of [9]). This comparison is required to adequately demonstrate the generic nature of the proposed methodology.

Results of Table II indicate that for the analysis of the IEEE 33 Bus Test System, the most appropriate equivalents are the TF-based model, the D-EXP(1) and D-EXP(2) models. Indeed, these models provide the most accurate estimates, presenting also high generalization capability. However, as demonstrated in [9], the fourth-order TF-based model cannot simulate accurately the dynamic behavior of the European medium voltage DN of CIGRE under high DG penetration levels, while the D-EXP(2) is not robust enough. In fact, for the analysis of the European medium voltage grid of CIGRE, the D-EXP(1) and D-ZIP(2) equivalents are more appropriate.

The comparison of the results reveals that the dynamic behavior of ADNs is affected by the grid topology, the location of the loads and DG units, and the interaction of the installed components. Therefore, the set of equivalent models required for the dynamic analysis of distinct DNs may differ.

Comparisons between Fig. 10 and Fig. 10 of [9] reveal that the DN topology, and thus the resulting dynamic responses used for parameter estimation, does not affect considerably the computational burden (execution time) of the examined models. Indeed, the computational burden is mainly affected by the total number of model parameters as well as by the structure of the equivalent.

## VII. Conclusions

In this paper, a methodology to assess the applicability of measurement-based equivalent models for the dynamic analysis of DNs is thoroughly evaluated. Towards this objective, 22 equivalent models are examined and their performance, in terms of accuracy and robustness, is assessed. Dynamic simulations were conducted in the IEEE 33 Bus Test System, assuming a wide range of DG penetration levels and operating conditions. Additionally, the most critical parameters of all examined equivalent models are identified and their importance is evaluated by means of a variance-based sensitivity analysis. Finally, the efficacy of the proposed variance-based sensitivity analysis is compared against conventional approaches.

Analysis reveals that DN characteristics, such as grid topology, DG penetration level, etc., affect significantly the dynamic behavior of the examined grid. Nevertheless, in all cases the proposed methodology identifies the most accurate and robust equivalent model, thus facilitating the dynamic equivalencing and dynamic analysis of modern DNs.

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