# A Robust Method for Transmission Line Sequence Parameter Estimation using Synchronised Phasor Measurements

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***Abstract*—We present a positive and zero sequence line parameter estimation method, that is robust to systematic errors in the instrument transformers, especially when they are within the specified tolerance as per standards. Using Monte Carlo simulations, it is shown that the proposed approach is robust and accurate for all operating conditions, specifically for short length and lightly loaded transmission lines. We also validate the proposed approach on 400 kV and 765 kV transmission lines using actual field phasor measurement unit (PMU) data. Further, estimation of zero sequence line parameter values is somewhat tricky because not enough unbalance exists during the normal operation of the power grid. Therefore, we quantify the minimum percentage unbalance needed in currents to determine zero sequence line parameter values within 1% tolerance. We also present the line length and line loading effects in estimating zero sequence line parameter values using simulations. Simulation and field PMU data results on 765 kV and 400 kV lines of different lengths and loading levels show that the proposed method estimates accurate positive and zero sequence line parameter values.**

***Index Terms*—Line parameter estimation, percentage unbalance, phasor measurement units (PMUs).**

## I. Introduction

TRANSMISSION line parameters are affected due to many factors like environmental conditions, inaccuracies in the line model, and mutual coupling of parallel transmission lines. Further, many power system studies like load flow, state estimation, distance protection, fault calculation, etc., require accurate knowledge of line parameters. In the past, the transmission line parameter values were determined from the tower geometry, conductor properties, conductor sag, actual line length estimates, etc., [1]. These computations are involved with approximations and assumptions such as uniform current density along the line, constant ground resistivity, temperature, constant material characteristics, etc., which may not give accurate line parameters. With phasor measurement units (PMUs) installed in the grid, the line parameter values can be calculated accurately.

Significant research on transmission line parameter estimation using PMU data has been reported in the past [2]–[5]. In these papers, typically linear least squares (LS) method is used in estimating line parameter values. Reference [6] proposed a recursive regression technique using a Kalman filter to determine three-phase line parameter values. However, this method assumes that there are no measurement errors in instrument transformers (ITs) associated with PMUs, which may not be true. Many publications [7]–[11] have reported the problem of estimation of IT ratio correction factors (RCFs) and transmission line parameters simultaneously. While the methods proposed in [7], [8] estimate the three-phase transmission parameters, the methods in [9]–[11] only estimate positive sequence impedance values. The authors in [10] also proposed tree based algorithm to describe the sequence of the transmission lines for parameter estimation. These references have defined RCF as the factor by which the nominal ratio as specified on the name plate of instrument transformer must be multiplied to obtain actual ratio.

Reference [12] proposed a combined algorithm of state estimation and parameter tracking to calculate the three-phase line parameter values of untransposed line using PMU measurements. In [12], the authors first employed three-phase static state estimation for the estimation of states. Then, using the Kalman filter, the parameters are tracked dynamically. Reference [13] developed non-linear algorithm based on extended Kalman filter to estimate line parameter values. The authors in [14] proposed maximum likelihood (ML) estimators which use noise covariance matrix to compute three-phase transmission line parameter values of an untransposed line. Reference [15] developed a linear regression method (M-estimator) to determine the three-phase line parameter values.

The zero sequence parameters are usually obtained inaccurately because of mutual coupling between transmission lines [16]. Further, the calculation of the zero sequence parameter is tricky compared to positive sequence parameter estimation as it requires enough zero sequence unbalance in the system [17]. To the best of our knowledge, only reference [18] has reported the quantitative evaluation of how a small percentage of unbalance is sufficient to determine zero sequence line parameter values accurately in prior literature. In [18], it is shown that the minimum percentage unbalance needed in estimating the zero sequence line parameter values accurately varies inversely with the line length and line loading level. Further, a method proposed in [18] assumes that there are no systematic errors in capacitive voltage transformers (CVTs) and current transformers (CTs) associated with PMUs, which need not be true. The systematic error in measurement is the difference between calculated and true value by a constant amount. The systematic errors in ITs affect transmission line parameter values. Hence, there is a need to study the sensitivity of systematic errors in the estimation of line parameter values.

In India, most of the ITs installed in the grid are of 0.5 class type. For 0.5 class metering CTs and CVTs, as per IEC standard 61869-2 and 61869-5 [19], [20], the permissible errors for magnitude correction factor (MCF) are $\pm 0.5\%$. For 0.5 class metering CTs and CVTs, the permissible errors for phase angle correction factor (PACF) are $\pm 30$ mins., and $\pm 20$ mins., respectively. Using Monte Carlo simulations, we will show that the existing methods of parameter estimation give erroneous line parameter values even when the systematic errors in ITs are within the permissible range as per [19], [20]. In this paper, we first present a robust method for positive sequence line parameter estimation. Next, we show how this proposed estimation method can be applied for computation of zero sequence impedance values. Further, we quantify the minimum amount of unbalance needed in currents to estimate zero sequence line parameter values reliably. We also show that the line loading level and line length do not affect the amount of unbalance needed to estimate zero sequence line parameter values using the proposed method.

Reference [2] presented a four methods comparison for determination of positive sequence transmission line parameter values. In [2], authors have incorporated biased and non-biased noise to study the line parameter accuracy for short transmission lines. Therefore, to assess the performance of the proposed approach, two best methods out of four methods from [2], and total least squares (TLS) approach which also models errors on the model matrix proposed in [3] are considered for comparison. These methods can be explained as follows:

1) ABCD Parameters Approach [2]:

The authors have suggested the ABCD Parameters approach. A transmission line can be described by the two-port parameters. Therefore, we have,

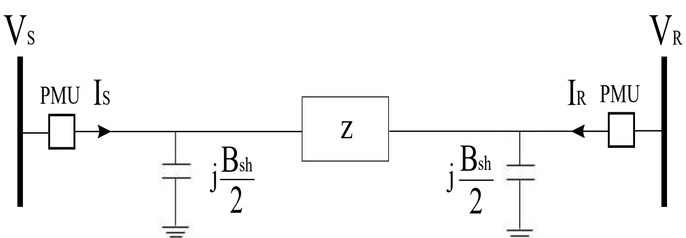


Fig. 1. Transmission line $\pi$-model.

$$\begin{aligned} \vec{V}_S &= \mathrm{A}\vec{V}_R + \mathrm{B}\vec{I}_R, \\ \vec{I}_S &= \mathrm{C}\vec{V}_R + \mathrm{D}\vec{I}_R. \end{aligned} \tag{1}$$

The above equations are complex equations. By expanding them into real equations with multiple time snapshots, we get,

$$\begin{bmatrix} \vec{V}^r_{S_1} \\ \vec{V}^i_{S_1} \\ \vdots \\ \vec{V}^r_{S_n} \\ \vec{V}^i_{S_n} \end{bmatrix} = \begin{bmatrix} \vec{V}^r_{R_1} & -\vec{V}^i_{R_1} & \vec{I}^r_{R_1} & -\vec{I}^i_{R_1} \\ \vec{V}^i_{R_1} & \vec{V}^r_{R_1} & \vec{I}^i_{R_1} & \vec{I}^r_{R_1} \\ \vdots & \vdots & \vdots & \vdots \\ \vec{V}^r_{R_n} & -\vec{V}^i_{R_n} & \vec{I}^r_{R_n} & -\vec{I}^i_{R_n} \\ \vec{V}^i_{R_n} & \vec{V}^r_{R_n} & \vec{I}^i_{R_n} & \vec{I}^r_{R_n} \end{bmatrix} \begin{bmatrix} \mathrm{A}^r \\ \mathrm{A}^i \\ \mathrm{B}^r \\ \mathrm{B}^i \end{bmatrix} \tag{2}$$

where $n$ is the number of independent measurements. From equation (2), A and B parameters can be determined by employing the least squares estimation technique. After calculating A and B, the series parameter $Z$ and shunt parameter $\dfrac{B_{sh}}{2}$ can be computed by,

$$Z = \mathrm{B}, \qquad \frac{B_{sh}}{2} = \frac{\mathrm{A}-1}{\mathrm{B}} \tag{3}$$

Henceforth, this method would be described as $M_1$.

2) Basic Approach [2]:

By applying Kirchhoff's voltage law (KVL) on two bus system, as shown in Fig. 1,

$$Z\vec{I}_S = \left(1 + jZ\frac{B_{sh}}{2}\right)\vec{V}_S - \vec{V}_R \tag{4}$$

$$Z\vec{I}_R = \left(1 + jZ\frac{B_{sh}}{2}\right)\vec{V}_R - \vec{V}_S \tag{5}$$

where $\vec{V}_S$, $\vec{V}_R$, $\vec{I}_S$, $\vec{I}_R$ are positive sequence phasors. Solving equations (4) and (5), the line parameters $Z$ and $\dfrac{B_{sh}}{2}$ can be obtained by,

$$Z = \frac{\vec{V}_S^2 - \vec{V}_R^2}{\vec{I}_S\vec{V}_R - \vec{I}_R\vec{V}_S}, \qquad \frac{B_{sh}}{2} = \frac{\vec{I}_S + \vec{I}_R}{\vec{V}_S + \vec{V}_R} \tag{6}$$

Henceforth, this method would be described as $M_2$.

3) Total Least Squares (TLS) Approach [3]:

Separating equations (4) and (5) into the real and imaginary parts,

$$\begin{bmatrix} (\vec{V}^r_S - \vec{V}^r_R) & -(\vec{V}^i_S - \vec{V}^i_R) & -\vec{V}^i_S \\ (\vec{V}^i_S - \vec{V}^i_R) & (\vec{V}^r_S - \vec{V}^r_R) & \vec{V}^r_S \\ (\vec{V}^r_R - \vec{V}^r_S) & -(\vec{V}^i_R - \vec{V}^i_S) & -\vec{V}^i_R \\ (\vec{V}^i_R - \vec{V}^i_S) & (\vec{V}^r_R - \vec{V}^r_S) & \vec{V}^r_R \end{bmatrix} \begin{bmatrix} G \\ B \\ \frac{B_{sh}}{2} \end{bmatrix} = \begin{bmatrix} \vec{I}^r_S \\ \vec{I}^i_S \\ \vec{I}^r_R \\ \vec{I}^i_R \end{bmatrix} \tag{7}$$

where $G + jB = \frac{1}{Z}$. Equation (7) can be written as below.

$$(\mathrm{A} + \mathrm{E})\mathrm{x} = \mathrm{b} + \mathrm{e}. \tag{8}$$

where model matrix A contains voltage measurements. The matirx E neutralizes the errors in voltage measurements of model matrix A. The vector x is $[G, B, \frac{B_{sh}}{2}]^{\mathrm{T}}$. b is the current measurement vector, and e represents the error in the current measurements. In this approach, the Frobenius norm of the matrix [E, e] is minimized. Therefore, the line parameter estimation (LPE) problem is formulated as,

$$\begin{aligned} &\min_{\mathrm{E,e,x}} ||(\mathrm{E} — \mathrm{e})||_F; \\ &\text{s.t. } (\mathrm{A} + \mathrm{E})\mathrm{x} = \mathrm{b} + \mathrm{e}. \end{aligned} \tag{9}$$

where $||.||_F$ is Frobenius norm. Here onwards, this method would be described as $M_3$.

The paper is organized as follows. Section II discusses a robust method for determination of positive sequence line parameter values and present the Monte Carlo simulations and field results. Section III deals with estimation of zero sequence line parameter values. Section IV presents the conclusion.

## II. Positive Sequence Line Parameter Estimation

Consider the problem of positive sequence line parameters estimation from PMU data. The equations that establish the relationship between current and voltage phasors of a transmission line are described as,

$$\begin{bmatrix}\vec{I}_S\\ \vec{I}_R\end{bmatrix} = \begin{bmatrix}Y_{bus}\end{bmatrix}\begin{bmatrix}\vec{V}_S\\ \vec{V}_R\end{bmatrix} \tag{10}$$

where,

$$\begin{bmatrix}Y_{bus}\end{bmatrix} = \begin{bmatrix}Y_{11} & Y_{12}\\ Y_{21} & Y_{22}\end{bmatrix}$$

If independent measurement sets for two different loading conditions are considered then, equation (10) can be written as,

$$\begin{bmatrix}\vec{I}_{S1} & \vec{I}_{S2}\\ \vec{I}_{R1} & \vec{I}_{R2}\end{bmatrix} = \begin{bmatrix}Y_{bus}\end{bmatrix}\begin{bmatrix}\vec{V}_{S1} & \vec{V}_{S2}\\ \vec{V}_{R1} & \vec{V}_{R2}\end{bmatrix}. \tag{11}$$

In (11), $\vec{I}_{S1}$, $\vec{I}_{R1}$, $\vec{I}_{S2}$, $\vec{I}_{R2}$, $\vec{V}_{S1}$, $\vec{V}_{R1}$, $\vec{V}_{S2}$, and $\vec{V}_{R2}$, are positive sequence current and voltage phasors related to two distinct loading levels. Hence, matrix $Y_{bus}$ can be calculated. After computing matrix $Y_{bus}$ using equation (11), series and shunt parameter values can be calculated as,

$$Z = R + jX = -\frac{1}{\text{mean}(Y_{12}, Y_{21})}, \quad \text{and}$$

$$j\frac{B_{sh}}{2} = \text{mean}\Big((Y_{11}+Y_{12}), \quad (Y_{22}+Y_{21})\Big). \tag{12}$$

We have not imposed symmetry required on matrix $Y_{bus}$ as a constraint. If that is imposed, the method becomes similar to method $M_2$. Hence, the mean of off-diagonal is taken as $Z$. Further, equation (11) can be rewritten as,

$$\begin{bmatrix}\vec{I}_{S1} & \vec{I}_{S2} & \cdots & \vec{I}_{Sn}\\ \vec{I}_{R1} & \vec{I}_{R2} & \cdots & \vec{I}_{Rn}\end{bmatrix} = \begin{bmatrix}Y_{bus}\end{bmatrix}\begin{bmatrix}\vec{V}_{S1} & \vec{V}_{S2} & \cdots & \vec{V}_{Sn}\\ \vec{V}_{R1} & \vec{V}_{R2} & \cdots & \vec{V}_{Rn}\end{bmatrix}. \tag{13}$$

where n is the number of independent measurements related to distinct loading conditions. The standard LS form can be obtained by transposing equation (13) with $\begin{bmatrix}Y_{bus}\end{bmatrix}^T = \begin{bmatrix}Y_{bus}\end{bmatrix}$.

$$\begin{bmatrix}\vec{I}_{S1} & \vec{I}_{R1}\\ \vec{I}_{S2} & \vec{I}_{R2}\\ \vdots & \vdots\\ \vec{I}_{Sn} & \vec{I}_{Rn}\end{bmatrix} = \begin{bmatrix}\vec{V}_{S1} & \vec{V}_{R1}\\ \vec{V}_{S2} & \vec{V}_{R2}\\ \vdots & \vdots\\ \vec{V}_{Sn} & \vec{V}_{Rn}\end{bmatrix}\begin{bmatrix}Y_{bus}\end{bmatrix}^T. \tag{14}$$

Hence, by employing the LS estimation technique in equation (14), the matrix $Y_{bus}$ can be calculated.

### A. Robustness of the Proposed Ybus Method

In comparison with other least squares (LS) formulations, like the ABCD parameter estimation approach (Method $M_1$) or TLS approach (Method $M_3$), and other methods like direct or basic approach (Method $M_2$), the proposed Ybus estimation approach is much more robust. This can be explained through formal error analysis as follows.
From equation (13),

$$\begin{bmatrix}\mathrm{I}\end{bmatrix}_{2\times n} = \begin{bmatrix}Y_{bus}\end{bmatrix}_{2\times 2}\begin{bmatrix}\mathrm{V}\end{bmatrix}_{2\times n}. \tag{15}$$

Since, rank(V) = 2, the number of rows, the Penrose Moore Inverse of matrix V is given by,

$$\mathrm{V}^+ = \mathrm{V}^H(\mathrm{V}\mathrm{V}^H)^{-1} \quad \text{and} \quad \mathrm{V}\mathrm{V}^+ = I$$

where $I$ is 2×2 identity matrix. Thus, from equation (15), we get,

$$\mathrm{I}\ \mathrm{V}^+ = \begin{bmatrix}Y_{bus}\end{bmatrix}$$

Suppose that current measurements are accurate. We can always check this from the small differential current in a short line.
Now, if $\vec{K}_{vs}$ and $\vec{K}_{vr}$ are RCFs for CVTs, we will estimate 2×2 $\begin{bmatrix}\hat{Y}_{bus}\end{bmatrix}$ instead of $\begin{bmatrix}Y_{bus}\end{bmatrix}$. Because $\mathrm{V}_{meas} = \dfrac{\mathrm{V}_{true}}{\mathrm{K}_v}$, the following equations are actually solved in the least squares (LS) sense. Therefore, from equation (15),

$$\mathrm{I} = \begin{bmatrix}\hat{Y}_{bus}\end{bmatrix}\mathrm{K}_v^{-1}\ \mathrm{V}, \quad \text{where} \ \ \mathrm{K}_v = \begin{bmatrix}\vec{K}_{vs} & 0\\ 0 & \vec{K}_{vr}\end{bmatrix}$$

Post multiplying by $\mathrm{V}^+$ on both the sides, we get,

$$\begin{bmatrix}Y_{bus}\end{bmatrix} = \mathrm{I}\ \mathrm{V}^+ = \begin{bmatrix}\hat{Y}_{bus}\end{bmatrix}\mathrm{K}_v^{-1} \tag{16}$$

$$\therefore \begin{bmatrix}\hat{Y}_{bus}\end{bmatrix} - \begin{bmatrix}Y_{bus}\end{bmatrix} = \begin{bmatrix}Y_{bus}\end{bmatrix}\begin{bmatrix}\mathrm{K}_v - I\end{bmatrix} \tag{17}$$

If we primarily focus on the impact of angle errors in CVTs, then,

$$\mathrm{K}_v = 1\ e^{j\Delta\delta} = \cos\Delta\delta + j\sin\Delta\delta = 1 + j\Delta\delta. \tag{18}$$

$$\therefore \begin{bmatrix}\mathrm{K}_v - I\end{bmatrix} = j\begin{bmatrix}\Delta\delta_{vs} & 0\\ 0 & \Delta\delta_{vr}\end{bmatrix}.$$

Therefore, from equation (17),

$$\begin{bmatrix}\Delta Y_{bus}\end{bmatrix} = \begin{bmatrix}\hat{Y}_{bus}\end{bmatrix} - \begin{bmatrix}Y_{bus}\end{bmatrix} = j\begin{bmatrix}Y_{bus}\end{bmatrix}\begin{bmatrix}\Delta\delta_{vs} & 0\\ 0 & \Delta\delta_{vr}\end{bmatrix}.$$

So, first important deduction is that error in $\begin{bmatrix}Y_{bus}\end{bmatrix}$ estimation method is only proportional to phase angle errors. They do not depend on the condition number of $\begin{bmatrix}Y_{bus}\end{bmatrix}$ and the errors will not increase quadratically, inversely, or exponentially with an increase in angle errors. Further, the proposed Ybus estimation approach outscores methods like $M_1$, $M_2$, and $M_3$. An important reason for it is the averaging, which can be explained as follows:
In our algorithm, we have,

$$Y_{line} = -\text{mean}\Big(Y_{bus}(1,2),\ Y_{bus}(2,1)\Big)$$

Theoretically, $Y_{bus}(1,2) = Y_{bus}(2,1)$ and there are not four parameters $\big(Y_{bus}(1,1),\ Y_{bus}(1,2),\ Y_{bus}(2,1),\ Y_{bus}(2,2)\big)$ to be estimated but only three ($R$, $X$, and $B_{sh}/2$). Many other approaches target this 3-parameter aspect. However, the averaging approach $Y_{line} = -\text{mean}\big(Y_{bus}(1,2),\ Y_{bus}(2,1)\big)$ is

extremely beneficial as follows.
Since, from equation (16), we have,

$$\left[\hat{Y}_{bus}\right] = \left[Y_{bus}\right]\begin{bmatrix}\vec{K}_{vs} & 0\\ 0 & \vec{K}_{vr}\end{bmatrix}.$$

$$\therefore -\hat{Y}_{bus}(1,2) = Y_{line}\ \vec{K}_{vr} \quad \text{and} \quad -\hat{Y}_{bus}(2,1) = Y_{line}\ \vec{K}_{vs}.$$

$$\therefore -\frac{\hat{Y}_{bus}(1,2)+\hat{Y}_{bus}(2,1)}{2} \;=\; Y_{line}\left(\frac{\vec{K}_{vs}+\vec{K}_{vr}}{2}\right).$$

Substituting from equation (18),

$$\hat{Y}_{line} \;=\; Y_{line}\left(1+j\,\frac{\Delta\delta_{vs}+\Delta\delta_{vr}}{2}\right).$$

$$\therefore Z_{line} \;=\; \hat{Z}_{line}\left(1+j\,\frac{\Delta\delta_{vs}+\Delta\delta_{vr}}{2}\right).$$

$$\therefore \frac{\hat{Z}_{line}-Z_{line}}{Z_{line}} \;=\; \left(1+j\,\frac{\Delta\delta_{vs}+\Delta\delta_{vr}}{2}\right)^{-1} -\ 1$$
$$\approx\ j\,\frac{\Delta\delta_{vs}+\Delta\delta_{vr}}{2}.$$

Now there are two possibilities, $\Delta\delta_{vs}$ and $\Delta\delta_{vr}$ having errors in same direction ($++$ or $--$) or in opposite direction ($+-$ or $-+$). In either case, the averaging reduces error in line impedance estimate. However, in the case of ($+-$) or ($-+$) scenario, due to cancellations, the averaging effect will increase accuracy tremendously. Hence, the Ybus estimation approach is robust vis-a-vis other approaches.

### *B. Sensitivity Analysis of Line Parameter Estimation using Monte Carlo Simulation*

For statistical performance evaluation of the proposed Ybus method, Monte Carlo simulations are performed. For a given line, measurements are received from six independent CVTs and CTs (both the ends three-phase voltages and currents). Each measurement of voltage and current has two independent elements, i.e., Amplitude and Angle. Therefore, to estimate line parameters, a total of 24 independent measurements are utilized. Systematic errors in any of these measurements will give erroneous line parameter estimates. For 0.5 class metering CT and CVT, a tolerable accuracy range as per IEC standards 61869-2 and 61869-5 [19], [20] is considered. A hundred thousand ($10^5$) simulations are carried out by randomly adding uniformly distributed bias errors within the permissible range for all 24 measurements.

Tables I, II, III, and IV show the comparison of the proposed method with methods $M_1$, $M_2$, and $M_3$ for both short and long transmission lines with variation of loading. It is observed that even with admissible IT systematic errors, with methods $M_2$ and $M_3$, the minimum and maximum series resistance and series reactance values nowhere near to design values for both short and long length lines. On the other hand, with method $M_1$, the minimum and maximum values for shunt susceptance are nowhere near to the design values for both the lines. However, in the case of the proposed approach, we can see that the minimum and maximum values for all three line impedances are very nearly to design values. Hence, methods $M_2$ and $M_3$ are not robust in estimating series line parameter values. While method $M_1$ can not robustly estimate shunt parameter values. However, the proposed method can robustly estimate both the series and shunt parameter values. Further, the standard deviation ($\sigma$) for all three line impedances is the minimum in the case of the proposed method.

From the results shown in Tables I and II, it can be observed that line parameter values estimated using methods $M_1$, $M_2$, and $M_3$ are more affected by systematic errors in IT for a short

TABLE I
ESTIMATED POSITIVE SEQUENCE PARAMETERS FOR A 400 KV, 50 KM, 100 MW LINE USING MONTE CARLO SIMULATION.

| | Error (%) = $\frac{\text{Estimated Value - Design Value}}{\text{Design Value}} \times 100$ | | | |
|---|---|---|---|---|
| | Proposed Method | $M_1$ [2] | $M_2$ [2] | $M_3$ [3] |
| $R_{mean}$ | 0.01 | -0.01 | 0.48 | 0.93 |
| $R_{max}$ | 11.46 | 14.23 | 765.10 | 644.34 |
| $R_{min}$ | -10.19 | -13.67 | -767.77 | -647.98 |
| $R_{\sigma}$ | 2.74 | 3.88 | 205.31 | 170.67 |
| $X_{mean}$ | 0 | 0 | -0.04 | 0.50 |
| $X_{max}$ | 0.64 | 0.93 | 76.31 | 65.17 |
| $X_{min}$ | -0.66 | -0.92 | -77.83 | -55.67 |
| $X_{\sigma}$ | 0.17 | 0.24 | 21.44 | 17.48 |
| $(B_{sh}/2)_{mean}$ | -0.001 | 0.43 | -0.01 | -0.01 |
| $(B_{sh}/2)_{max}$ | 2.98 | 602.63 | 7.57 | 7.60 |
| $(B_{sh}/2)_{min}$ | -3.24 | -585.90 | -7.16 | -7.19 |
| $(B_{sh}/2)_{\sigma}$ | 0.46 | 162.86 | 2.01 | 2.02 |

Design value: $R$ = 1.405 Ω, $X$ = 15.42 Ω, $\frac{B_{sh}}{2}$ = 9.456×$10^{-5}$ ℧

TABLE II
ESTIMATED POSITIVE SEQUENCE PARAMETERS FOR A 400 KV, 200 KM, 100 MW LINE USING MONTE CARLO SIMULATION.

| | Error (%) | | | |
|---|---|---|---|---|
| | Proposed Method | $M_1$ [2] | $M_2$ [2] | $M_3$ [3] |
| $R_{mean}$ | 0.01 | -0.01 | 0.15 | 0.15 |
| $R_{max}$ | 11.46 | 14.23 | 222.92 | 218.41 |
| $R_{min}$ | -10.18 | -13.67 | -222.34 | -217.99 |
| $R_{\sigma}$ | 2.74 | 3.88 | 60.18 | 58.93 |
| $X_{mean}$ | 0 | 0 | -0.01 | -0.01 |
| $X_{max}$ | 0.64 | 0.93 | 22.33 | 21.90 |
| $X_{min}$ | -0.66 | -0.92 | -22.84 | -22.30 |
| $X_{\sigma}$ | 0.17 | 0.24 | 6.28 | 6.15 |
| $(B_{sh}/2)_{mean}$ | -0.001 | 0.03 | -0.003 | -0.003 |
| $(B_{sh}/2)_{max}$ | 0.68 | 37.27 | 1.81 | 1.81 |
| $(B_{sh}/2)_{min}$ | -0.65 | -36.44 | -1.63 | -1.63 |
| $(B_{sh}/2)_{\sigma}$ | 0.17 | 10.07 | 0.42 | 0.42 |

Design value: $R$ = 5.619 Ω, $X$ = 61.68 Ω, $\frac{B_{sh}}{2}$ = 3.782×$10^{-4}$ ℧

TABLE III
ESTIMATED POSITIVE SEQUENCE PARAMETERS FOR A 400 KV, 50 KM, 400 MW LINE USING MONTE CARLO SIMULATION.

| | Error (%) | | | |
|---|---|---|---|---|
| | Proposed Method | $M_1$ [2] | $M_2$ [2] | $M_3$ [3] |
| $R_{mean}$ | 0.01 | -0.01 | 0.15 | 0.15 |
| $R_{max}$ | 11.46 | 14.23 | 227.55 | 222.89 |
| $R_{min}$ | -10.18 | -13.67 | -225.63 | -220.84 |
| $R_{\sigma}$ | 2.74 | 3.88 | 61.09 | 59.81 |
| $X_{mean}$ | 0 | 0 | -0.01 | -0.01 |
| $X_{max}$ | 0.64 | 0.93 | 22.69 | 22.25 |
| $X_{min}$ | -0.66 | -0.92 | -23.10 | -22.55 |
| $X_{\sigma}$ | 0.17 | 0.24 | 6.38 | 6.24 |
| $(B_{sh}/2)_{mean}$ | -0.001 | 0.43 | -0.03 | -0.03 |
| $(B_{sh}/2)_{max}$ | 2.98 | 602.63 | 22.95 | 22.98 |
| $(B_{sh}/2)_{min}$ | -3.24 | -585.90 | -22.20 | -22.23 |
| $(B_{sh}/2)_{\sigma}$ | 0.46 | 162.86 | 6.17 | 6.18 |

Design value: $R$ = 1.405 Ω, $X$ = 15.42 Ω, $\frac{B_{sh}}{2}$ = 9.456×$10^{-5}$ ℧

TABLE IV
ESTIMATED POSITIVE SEQUENCE PARAMETERS FOR A 400 KV, 200 KM, 400 MW LINE USING MONTE CARLO SIMULATION.

| | Error (%) | | | |
|---|---|---|---|---|
| | Proposed Method | $M_1$ [2] | $M_2$ [2] | $M_3$ [3] |
| $R_{mean}$ | 0.01 | -0.01 | 0.05 | 0.05 |
| $R_{max}$ | 11.46 | 14.23 | 63.88 | 63.78 |
| $R_{min}$ | -10.18 | -13.67 | -60.61 | -60.61 |
| $R_{\sigma}$ | 2.74 | 3.88 | 16.30 | 16.27 |
| $X_{mean}$ | 0 | 0 | -0.002 | -0.002 |
| $X_{max}$ | 0.64 | 0.93 | 6.04 | 6.03 |
| $X_{min}$ | -0.66 | -0.92 | -6.14 | -6.13 |
| $X_{\sigma}$ | 0.17 | 0.24 | 1.69 | 1.68 |
| $(B_{sh}/2)_{mean}$ | -0.001 | 0.03 | -0.01 | -0.01 |
| $(B_{sh}/2)_{max}$ | 0.68 | 37.27 | 5.42 | 5.42 |
| $(B_{sh}/2)_{min}$ | -0.65 | -36.44 | -5.09 | -5.09 |
| $(B_{sh}/2)_{\sigma}$ | 0.17 | 10.07 | 1.43 | 1.43 |

Design value: $R$ = 5.619 Ω, $X$ = 61.68 Ω, $\frac{B_{sh}}{2}$ = 3.782×$10^{-4}$ ℧

length line in comparison to long length line. From Tables I and III, we see that line parameters estimated with the methods $M_2$ and $M_3$ are also affected by loading of the line. Hence, methods $M_2$ and $M_3$ are sensitive to both loading and length of the line. While method $M_1$ is sensitive to the length of the line. However, the proposed method is not affected by both loading and length of the line.

From the results shown in Table V, we see that the line parameter values determined with methods $M_1$, $M_2$, and $M_3$ are more sensitive to IT errors. Even with a 1% bias error in magnitude of voltage of one end CVT, methods $M_2$ and $M_3$ fail in estimating accurate series resistance value. On the other hand, method $M_1$ fails to estimate accurate shunt parameter value. Further, it can be seen that the percentage errors in measurement of resistance and reactance are less with the the proposed method in comparison to method $M_1$. This leads to superior performance of the proposed Ybus approach in comparison to methods $M_1$, $M_2$, and $M_3$.

TABLE V
PERCENTAGE ERRORS IN THE LINE PARAMETER ESTIMATION WITH 1% SYSTEMATIC ERROR IN MAGNITUDE OF VOLTAGE FOR A 400 KV, 200 KM, 180 MW LINE.

| Line Parameter | Error (%) | | | |
|---|---|---|---|---|
| | Proposed Method | $M_1$ [2] | $M_2$ [2] | $M_3$ [3] |
| $R$ | 0.5 | 1.01 | -119.26 | -118.65 |
| $X$ | 0.5 | 1.01 | 1.50 | 1.50 |
| $B_{sh}/2$ | -0.5 | -42.51 | -0.5 | -0.5 |

## C. Field Results

The proposed Ybus formulation is also implemented over real field PMU data for a 400 kV, 83 km line and a 765 kV, 230 km line in India. The percentage error in the determination of positive sequence transmission line parameters for both the lines is shown in Tables VI and VII. It can be seen that the line parameters estimated in the case of the proposed approach are very close to the design values in comparison to methods $M_1$ $M_2$ and $M_3$. From the results of Tables VI and VII, it is observed that methods $M_2$ and $M_3$ estimate wrong series resistance values. On the other hand, method $M_1$ fails to estimate the shunt parameter value accurately. Further, we see that for both the lines, with method $M_1$, the percentage errors for determining the series resistance and reactance parameter are large compared to the proposed method. Therefore, the proposed Ybus approach can determines both the line series and shunt parameters accurately for both lines.

TABLE VI
PERCENTAGE ERRORS IN ESTIMATING LINE IMPEDANCE VALUES FOR A 400 KV, 83 KM, 100 MW LINE USING 1 HOUR ACTUAL PMU DATA.

| Line Parameter | Error (%) | | | |
|---|---|---|---|---|
| | Proposed Method | $M_1$ [2] | $M_2$ [2] | $M_3$ [3] |
| $R$ | -6.17 | -16.81 | -162,57 | -162.70 |
| $X$ | -1.55 | -10.49 | 4.68 | 4.64 |
| $B_{sh}/2$ | 5.86 | 94.69 | 6.56 | 6.56 |

TABLE VII
PERCENTAGE ERRORS IN ESTIMATING LINE IMPEDANCE VALUES FOR A 765 KV, 230 KM, 50 MW LINE USING 40 MINUTES ACTUAL PMU DATA.

| Line Parameter | Error (%) | | | |
|---|---|---|---|---|
| | Proposed Method | $M_1$ [2] | $M_2$ [2] | $M_3$ [3] |
| $R$ | 3.04 | 22.94 | 739.96 | 741.24 |
| $X$ | 0.95 | -4.86 | 13.84 | 13.54 |
| $B_{sh}/2$ | 4.15 | 15.22 | 4.09 | 4.09 |

## III. Zero Sequence Line Parameter Estimation

We now discuss the problem of zero sequence line parameter estimation from PMU data. The equation (10) in the phase domain can be written as,

$$\begin{bmatrix} \vec{I}_S^{abc} \\ \vec{I}_R^{abc} \end{bmatrix} = \begin{bmatrix} Y_{bus}^{abc} \end{bmatrix} \begin{bmatrix} \vec{V}_S^{abc} \\ \vec{V}_R^{abc} \end{bmatrix} \tag{19}$$

where,

$$\begin{bmatrix} Y_{bus}^{abc} \end{bmatrix} = \begin{bmatrix} Y_{11}^{abc} & Y_{12}^{abc} \\ Y_{21}^{abc} & Y_{22}^{abc} \end{bmatrix}$$

$$\vec{V}_S^{abc} = \begin{bmatrix} \vec{V}_S^a, \vec{V}_S^b, \vec{V}_S^c \end{bmatrix}^T, \vec{I}_S^{abc} = \begin{bmatrix} \vec{I}_S^a, \vec{I}_S^b, \vec{I}_S^c \end{bmatrix}^T,$$
$$\vec{V}_R^{abc} = \begin{bmatrix} \vec{V}_R^a, \vec{V}_R^b, \vec{V}_R^c \end{bmatrix}^T, \vec{I}_R^{abc} = \begin{bmatrix} \vec{I}_R^a, \vec{I}_R^b, \vec{I}_R^c \end{bmatrix}^T.$$

Here, matrix $\begin{bmatrix} Y_{bus}^{abc} \end{bmatrix}$ be the $6 \times 6$ matrix. The matrices $Y_{11}^{abc}$, $Y_{12}^{abc}$, $Y_{21}^{abc}$, and $Y_{22}^{abc}$ be the $3 \times 3$ matrices.

By considering independent measurement sets for two different loading conditions, the equation (19) in the matrix form becomes,

$$\underbrace{\begin{bmatrix} \vec{I}_{S1}^{abc} & \vec{I}_{S2}^{abc} \\ \vec{I}_{R1}^{abc} & \vec{I}_{R2}^{abc} \end{bmatrix}}_{6 \times 2} = \underbrace{\begin{bmatrix} Y_{bus}^{abc} \end{bmatrix}}_{6 \times 6} \underbrace{\begin{bmatrix} \vec{V}_{S1}^{abc} & \vec{V}_{S2}^{abc} \\ \vec{V}_{R1}^{abc} & \vec{V}_{R2}^{abc} \end{bmatrix}}_{6 \times 2}. \tag{20}$$

Further, equation (20) can be extended as,

$$\underbrace{\begin{bmatrix} \vec{I}_{S1}^{abc} & \vec{I}_{S2}^{abc} & \cdots & \vec{I}_{Sn}^{abc} \\ \vec{I}_{R1}^{abc} & \vec{I}_{R2}^{abc} & \cdots & \vec{I}_{Rn}^{abc} \end{bmatrix}}_{6 \times \text{n}} = \underbrace{\begin{bmatrix} Y_{bus}^{abc} \end{bmatrix}}_{6 \times 6} \underbrace{\begin{bmatrix} \vec{V}_{S1}^{abc} & \vec{V}_{S2}^{abc} & \cdots & \vec{V}_{Sn}^{abc} \\ \vec{V}_{R1}^{abc} & \vec{V}_{R2}^{abc} & \cdots & \vec{V}_{Rn}^{abc} \end{bmatrix}}_{6 \times \text{n}}. \tag{21}$$

where n is the number of independent measurements related to different loading conditions. Hence, from equation (21), matrix $Y_{bus}^{abc}$ can be determined using LS technique. After calculating matrix $Y_{bus}^{abc}$ using equation (21), series and shunt parameter matrices can be determined as,

$$\begin{aligned} Z^{abc} &= -\left( \text{mean}(Y_{12}^{abc}, Y_{21}^{abc}) \right)^{-1}, \quad \text{and} \\ j\frac{B_{sh}^{abc}}{2} &= \text{mean}\left( (Y_{11}^{abc} + Y_{12}^{abc}), \quad (Y_{22}^{abc} + Y_{21}^{abc}) \right). \end{aligned} \tag{22}$$

The sequence series impedance and shunt susceptance matrices ($Z^{seq}$, $B_{sh}^{seq}$) can be calculated as [1],

$$Z^{seq} = T^{-1}\ Z^{abc}\ T \quad \text{and} \quad B_{sh}^{seq} = T^{-1}\ B_{sh}^{abc}\ T. \tag{23}$$

In equation (23), T represents the sequence transformation matrix.

### A. Simulation Results

For our case study, a two-bus system with an untransposed line is modelled in ATP-EMTP software. LCC (Line constants, Cable constants, and Cable parameters) component in ATP-EMTP is considered to model untransposed lines of 220 kV, 400 kV, and 765 kV. To develop tangible theory, we had to assume transposition, which leads to decoupling of the symmetrical components. However, short and medium lines may not be transposed. To assess the impact of the assumption of transposition on the performance of the method, the simulation modelled a more realistic untransposed line.

From instantaneous voltage and current measurements generated from the ATP-EMTP simulation, the synchrophasors are calculated by applying Discrete Fourier Transform (DFT) technique. The unbalance is generated by varying the three phase voltage magnitude (one phase at a time) from the rated value at both ends of the line. The ratio of zero sequence current ($I_0$) to positive sequence current ($I_1$) is taken as a measure of the unbalance [17].

Tables VIII shows the percentage error to determine zero sequence parameter values for 220 kV, 400 kV, and 765 kV lines with the proposed Ybus formulation. It can be observed that the proposed approach can estimate the zero sequence line parameter values accurately. Table VIII also shows the minimum required percentage unbalance ($|I_0/I_1|$) to calculate zero sequence line impedance values at surge impedance loading (SIL). It is seen that the required minimum amount of unbalance was less than $2\%$ to estimate zero sequence line parameter values accurately. It is observed that if the percentage unbalance is less than the specified, the percentage error to estimate zero sequence line parameter values increases.

TABLE VIII
MINIMUM AMOUNT OF UNBALANCE NEEDED TO ESTIMATE ZERO SEQUENCE LINE PARAMETER VALUES FOR LESS THAN $1\%$ ERROR AT SIL USING PROPOSED YBUS METHOD

| Line Details | $\left\lvert \frac{I_0}{I_1} \right\rvert$ (%) | Error (%) | | |
|---|---|---|---|---|
| | | $R$ | $X$ | $B_{sh}/2$ |
| 220 kV, 150 km, 140 MW | 1.18 | 0.31 | 0.50 | -0.35 |
| 400 kV, 200 km, 520 MW | 1.52 | 0.45 | 0.34 | -0.24 |
| 765 kV, 300 km, 2275 MW | 1.25 | 0.34 | 0.31 | -0.47 |

Further simulations are performed to ascertain the line length and line loading effects in estimating zero sequence line parameter values. Let the method described in [18] be named as $M_4$. Fig. 2 shows the comparison of the proposed Ybus approach with method $M_4$ for a 400 kV, 100 MW line with various line lengths. It is observed that with method $M_4$, the percentage unbalance needed for the determination of zero sequence line parameter values decreases with an increase in line length. However, in the case of the proposed method, the percentage unbalance needed for the calculation of zero sequence line parameter values remains nearly constant irrespective of line length.

The minimum required percentage unbalance to calculate zero sequence parameter values of a 400 kV, 200 km line for different line loading levels is shown in Fig. 3. It is seen that with method $M_4$, the percentage unbalance needed for the computation of zero sequence line parameter values decreases with an increase in line loading. However, in the case of the proposed method, the percentage unbalance needed to

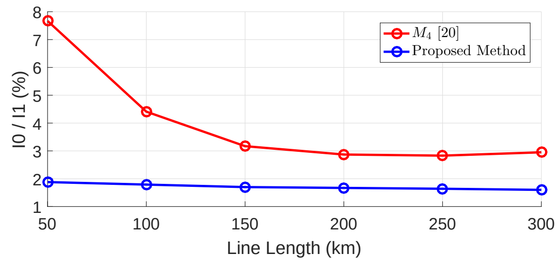


Fig. 2. Comparison of minimum amount of percentage unbalance needed to compute zero sequence line impedance values for different lengths of a 400 kV, 100 MW line.

calculate zero sequence line parameter values remains nearly constant irrespective of line loading level. We thus conclude that the performance of the proposed approach is not affected by transmission line length and loading level. Hence, the proposed Ybus approach is robust for both positive and zero sequence line parameter estimation.

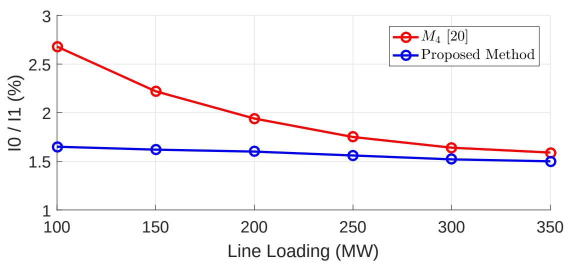


Fig. 3. Comparison of minimum amount of percentage unbalance needed to compute zero sequence line impedance values for different loading levels of a 400 kV, 200 km line.

## IV. Conclusion

In this paper, we proposed a robust method of positive and zero sequence line parameter estimation that works for a wide range of operating conditions, the most challenging ones being short lines with low loading. The proposed method is simple and easy to implement as it uses a Ybus matrix of a two-terminal line. Further, our field work with utilities in India has shown that practically all existing line parameter estimation methods fail for short and lightly loaded lines. Using Monte Carlo simulations, it was shown that even if small systematic errors are present in both end ITs, its cumulative effect on a short and lightly loaded line can lead to bizarre resistance estimates. However, the proposed method is robust to biasing errors in the ITs.

Further, we observed that the estimation of the zero sequence parameter is difficult compared to positive sequence parameter estimation as it requires enough zero sequence excitation. Therefore, we quantified empirically the minimum percentage unbalance needed in currents for the determination of zero sequence line impedance values within $1\%$ error. We also discussed the line length and line loading effects in estimating zero sequence line parameter values using proposed Ybus method. It was shown that the minimum amount of unbalance needed to calculate the zero sequence line parameter values using the proposed Ybus approach does not depend on line length and line loading level. Simulation and actual field results for 220 kV, 400 kV, and 765 kV lines show the efficacy of the proposed Ybus approach.

## Appendix A

The specification for the transmission line model used for this study is as follows:
Using LCC (Line constants, Cable constants, and Cable parameters) component in ATP-EMTP, single circuit, untransposed, bundle conductor line of 400 kV, 200 km was modelled. LCC subroutine calculates impedance parameters of line by providing its geometry. The line geometry is as follows:
Conductor diameter = 31.77 mm, bundle spacing = 45 cm, phase to phase spacing = 8 m, ground clearance = 12 m.


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