Small-Argument Analytical Expressions for the Calculation of the Ground-Return Impedance and Admittance of Underground Cables

Alberto De Conti, Naiara Duarte, Rafael Alipio, Osis E. S. Leal

Abstract—**This paper proposes small-argument approximations for two closed-form equations that were recently derived for the calculation of the ground-return impedance and admittance of underground cables. The proposed expressions are shown to be accurate up to 1 MHz for a typical cable configuration and frequency-dependent ground parameters. Their accuracy is also demonstrated in the calculation of transients on underground cables taking as reference results obtained with more general formulations.**

Keywords—**Electromagnetic transients, analytical expressions, ground-return admittance, ground-return impedance, underground cables.**

I. Introduction

THE calculation of the per-unit-length parameters of cable systems has been attracting a lot of attention in the last decades [1]–[5]. Recent efforts have been often directed to the derivation of rigorous expressions for determining the ground-return impedance and admittance of underground cables [6]–[8]. However, the obtained expressions are cast in terms of improper integrals whose evaluation is not straightforward. This has been the prime motivation for the derivation of simplified analytical expressions that could serve as an alternative to these integral equations within certain limits [9]–[12].

One of the most promising integral equations proposed in the recent years for the calculation of the ground-return impedance and admittance of underground cables was introduced by Xue *et al.* [8]. Their validity was demonstrated both in the frequency and time domains through comparisons with full-wave electromagnetic models [13]–[15]. However, the calculation of the ground-return impedance and admittance with Xue *et al.*'s equations in their original form is not straightforward because three improper integrals need to be evaluated.

In a recent paper, De Conti *et al.* [16] proposed a pair of closed-form expressions that reproduce the behavior of the equations of Xue *et al.* [8] with great accuracy up to 10 MHz for typical cable separations and a wide range of ground resistivities. However, no extensive transient simulations were performed to demonstrate the accuracy of these newly proposed expressions in the time domain. Moreover, although the proposed expressions significantly simplify the calculation of the ground-return impedance and admittance compared to Xue *et al.*'s equations [8], they still depend on the computation of Bessel functions that can be unfriendly for practitioners and cannot be easily evaluated in a scientific calculator.

This paper proposes an additional simplification to the closed-form expressions introduced in [16] by adopting a small-argument approximation to the Bessel functions appearing in these expressions. The obtained small-argument equations are shown to be accurate up to 1 MHz for different values of ground resistivity, which covers the frequency range of most transient phenomena affecting underground cable systems. The validity of the proposed equations is demonstrated through the calculation of transients on a typical 138-kV cable configuration. The simulations also demonstrate the accuracy of the closed-form expressions proposed in [16] in the simulation of electromagnetic transients.

This paper is organized as follows. Section II introduces the proposed small-argument approximations. Section III demonstrates the validity of the proposed equations in the frequency domain. Time-domain results are presented in Section IV, followed by conclusions in Section V.

II. Proposed Equations

A. Closed-form Approximations proposed in [16]

By performing suitable approximations to the integral equations of Xue *et al.* [8], De Conti *et al.* [16] derived the following closed-form expression for calculating the ground-return impedance of underground cables

$$Z_{g(m,n)} = \frac{j\omega\mu_0}{2\pi}\left[K_0\left(\gamma_1 d\right) + \frac{(\gamma_1-\gamma_0)}{(\gamma_0+\gamma_1)}e^{-(h_m+h_n)\gamma_1}\left(\frac{2}{4+\gamma_1^2 r^2}\right)\right], \quad (1)$$

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where ω is the angular frequency, μ_0 is the vacuum permeability, K_0 is the modified Bessel function of order zero, $h_m > 0$ and $h_n > 0$ correspond to the burial depths of cables m and n, respectively, r is the horizontal cable separation, measured from center to center, $d = \sqrt{(h_m - h_n)^2 + r^2}$, and γ_0 and γ_1 are the intrinsic propagation constants of the air and the ground, given by

$$\gamma_0 = j\omega\sqrt{\mu_0\varepsilon_0} \tag{2}$$

$$\gamma_1 = \sqrt{j\omega\mu_1\left(\sigma_1 + j\omega\varepsilon_1\right)} \tag{3}$$

where ε_0 is the vacuum permittivity, $\mu_1 = \mu_0$ is the ground permeability, σ_1 is the ground conductivity and ε_1 is the ground permittivity. Equation (1) can be used to directly determine the off-diagonal terms of the ground-return impedance matrix of a system of underground cables. For calculating the main-diagonal term $Z_{g(m,m)}$, r must be replaced by the total radius of cable m in (1), including the external jacket, and $h_n = h_m$ is assumed.

For calculating the potential coefficients required for determining the ground-return admittance matrix, De Conti *et al.* [16] proposed the following equation after performing approximations to the integral equations of Xue *et al.* [8]

$$P_{g(m,n)} = \frac{j\omega}{2\pi\left(\sigma_1 + j\omega\varepsilon_1\right)}\left[K_0\left(\gamma_1 d\right) + \alpha K_0\left(\gamma_1 D\right)\right] \tag{4}$$

where

$$\alpha = \frac{\gamma_1^2 - \gamma_0^2}{\gamma_1^2 + \gamma_0^2} \tag{5}$$

and $D = \sqrt{(h_m + h_n)^2 + r^2}$.

After using (4) to assemble matrix $\boldsymbol{P_g}$, the ground-return admittance matrix is determined as

$$\boldsymbol{Y_g} = j\omega\boldsymbol{P_g}^{-1} \tag{6}$$

Equations (1) and (4) were shown in [16] to reproduce Xue *et al.'s* equations [8] in a wide frequency range for ground resistivities ranging from 100 to 10000 Ωm and typical cable separations considering horizontal, vertical and trefoil cable configurations. Both equations are taken as reference for obtaining the approximations introduced in the next subsection. Although the ground-return admittance has been historically dismissed in the calculation of the ground-return parameters of underground cables, it is shown in [17] that this parameter should not be neglected for frequencies above some tens of kHz, for low-resistivity soils, and that this frequency limit is expected to reduce as the ground resistivity increases.

B. Small-Argument Approximations

By using the small-argument representation [18]

$$K_0\left(z\right) \approx -\ln\left(\frac{z}{2}\right) - \gamma \tag{7}$$

where $\gamma = 0.5772\ldots$ is the Euler-Mascheroni constant and $0 < z \ll 1$, equations (1) and (4) can be respectively simplified to

$$Z_{g(m,n)} = \frac{j\omega\mu_0}{2\pi}\left[-\ln\left(\frac{\gamma_1 d}{2}\right) - \gamma + \frac{(\gamma_1 - \gamma_0)}{(\gamma_0 + \gamma_1)}e^{-(h_m + h_n)\gamma_1}\left(\frac{2}{4 + \gamma_1^2 r^2}\right)\right] \tag{8}$$

and

$$P_{g(m,n)} = \frac{j\omega}{2\pi\left(\sigma_1 + j\omega\varepsilon_1\right)}\left\{\ln\left(\frac{D}{d}\right) - (\alpha + 1)\left[\gamma + \ln\left(\frac{\gamma_1 D}{2}\right)\right]\right\}. \tag{9}$$

Equations (8) and (9) have the advantage over (1) and (4) of avoiding the calculation of Bessel functions, which make them easily evaluated in any scientific calculator. However, it is necessary to investigate whether these equations are sufficiently accurate for the calculation of the ground-return parameters of underground cables. This is performed in the next couple of sections.

III. Frequency-Domain Analysis

To investigate the validity of the small-argument approximations (8) and (9) in the frequency domain, the 138-kV cable configuration shown in Fig. 1 is considered. Details of the cable properties and dimensions are given in Table I. For calculating the ground-return impedance and admittance of the cable, the realistic soil model proposed by Alipio and Visacro [19] is considered. This model predicts the frequency dependence of the ground conductivity and permittivity with the following equations

$$\sigma_1 = \sigma + 4.68 \times 10^{-6}\sigma^{0.27}f^{0.54} \tag{10}$$

$$\varepsilon_1 = 12\varepsilon_0 + 9.54 \times 10^{4}\sigma^{0.27}f^{-0.46}\varepsilon_0 \tag{11}$$

where σ is the low-frequency ground conductivity, in S/m, which is related to the low-frequency ground resistivity ρ as $\sigma = 1/\rho$, and f is the frequency, in Hz. Equation (10) is a semi-theoretical model that describes the mean variation of the ground conductivity with frequency based on *in situ* measurements of the frequency response of this parameter for 65 different types of soil with resistivity values ranging from 60 to 18,000 Ωm. Equation (11) is derived from (10) using the KramersKronigs relations to assure model causality. The validity of (10) and (11) was strictly demonstrated for frequencies below 4 MHz [19], but due to their physical consistency they have been often applied in a wider frequency range [20], [21]. Here, they are assumed to be valid up to 10 MHz. Equations (10) and (11) are recommended by the Cigré for including the frequency dependence of the soil parameters in electromagnetic modeling [20].

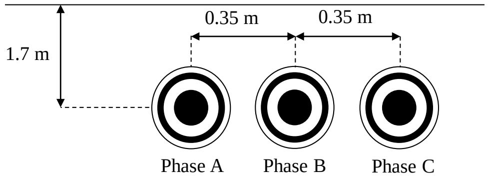

Fig. 1. 138-kV underground cable system.

TABLE I
CABLE PARAMETERS

Parameter	Core	Sheath
Internal radius (m)	0	0.032315
External radius (m)	0.012975	0.035315
Resistivity (Ωm)	1.93×10^{-8}	2.20×10^{-7}
Relative permeability	1	1
Relative permittivity of insulation	2.963538	2.3
Total cable radius (m)	0.039315	

A. Ground-Return Impedance

Fig. 2 illustrates the relative error in the calculation of the self-term of the ground-return impedance matrix, $Z_{g(1,1)}$, considering the closed-form expression (1) and the small-argument approximation (8), taking as reference Xue *et al.*'s integral equation [8]. The calculations were performed for $\rho = 100$, 1000, and 10000 Ωm considering the soil model of Alipio and Visacro [19]. The results are shown for frequencies greater than 100 Hz because for lower frequencies the errors are negligible. Although the validity of Xue *et al.*'s equations has not been demonstrated for a ground resistivity of 10000 Ωm in the high-frequency range, this value was considered for illustrating the ability of the proposed expressions to reproduce their behavior even for an extreme value of ground resistivity and consequently all values in between. It is observed in Fig. 2 that (1) and (8) lead to equivalent results regardless of ground resistivity, and that the deviations with regard to Xue *et al.*'s equations do not exceed 2% neither in the absolute value nor in the phase angle of $Z_{g(1,1)}$. Since the three cables are equal, $Z_{g(1,1)} = Z_{g(2,2)} = Z_{g(3,3)}$.

Fig. 3 shows the relative error in the calculation of the mutual term $Z_{g(3,1)}$ of the ground-return impedance matrix, which relates phases A and C (see Fig. 1). The calculations were again performed with (1) and (8), taking as reference Xue *et al.*'s integral equation [8]. This time, the relative errors obtained with (1) are slightly larger than the ones previously obtained for $Z_{g(1,1)}$. This result was already expected because, as shown in [16], equation (1) gradually loses accuracy in the high-frequency range as the cable separation increases [16]. Even so, the relative errors due to (1) are mostly within 5%, except for the phase angle associated $\rho = 1000$ Ωm, which exceeds 7% above 8 MHz. Interestingly, the small-argument approximation (8) presents a performance comparable to (1) up to the 1-2 MHz range, regardless of ground resistivity, which covers most transient phenomena affecting cable systems. The errors associated with the calculation of $Z_{g(2,1)}$, which is the mutual ground-return impedance between phases A and B (and, equivalently, between phases B and C) are

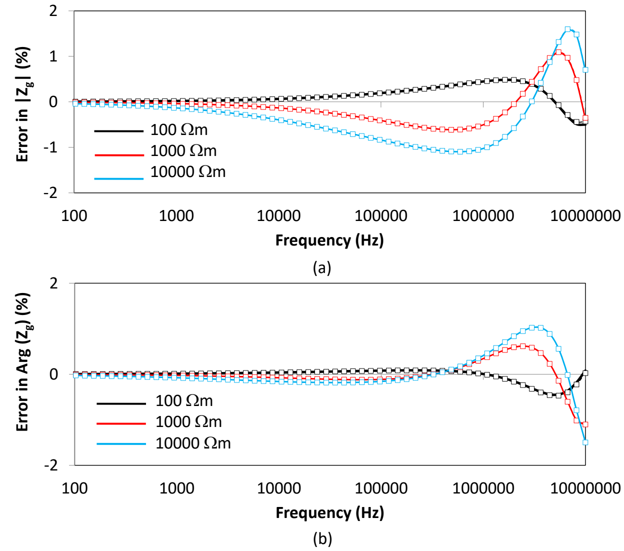

Fig. 2. Relative error in the calculation of element (1,1) of the ground-return impedance matrix taking as reference Xue *et al.*'s integral equation [8]. Solid lines: closed-form expression (1); dashed-lines with markers: small-argument expression (8). The results obtained with both equations are indistinguishable.

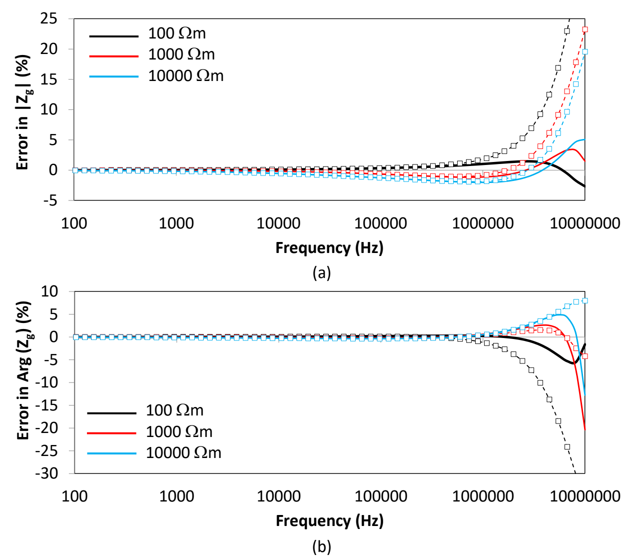

Fig. 3. Relative error in the calculation of element (3,1) of the ground-return impedance matrix taking as reference Xue *et al.*'s integral equation [8]. Solid lines: closed-form expression (1); dashed-lines with markers: small-argument expression (8).

greater than those observed for $Z_{g(1,1)}$ (see Fig. 2), but lower than those shown in Fig. 3 for $Z_{g(3,1)}$. For this reason, they are not shown here.

B. Ground-Return Potential Coefficients

Fig. 4 shows the relative error in the calculation of the self-term $P_{g(1,1)}$ of the ground-return potential coefficient matrix calculated with (4) and the small-argument approximation (9). For calculating the error in the absolute value of $P_{g(1,1)}$, the corresponding value obtained from Xue *et al.*'s integral equation [8] was taken as reference at each frequency sample. For determining the error in the argument

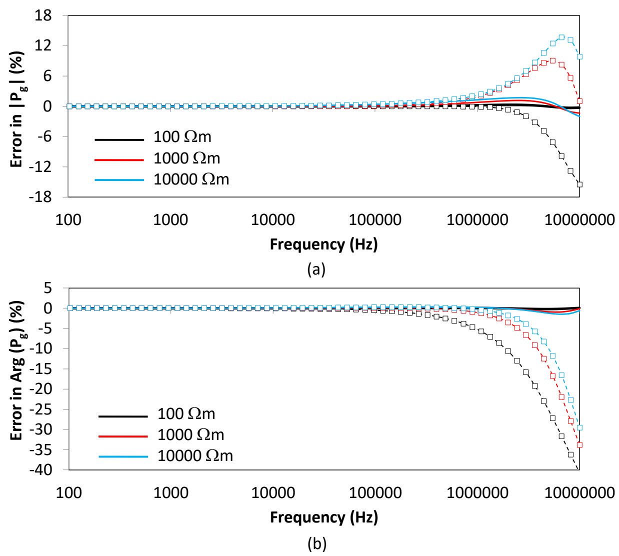

Fig. 4. Relative error in the calculation of element (1,1) of the ground-return potential coefficients matrix taking as reference Xue *et al.*'s integral equation [8]. Solid lines: closed-form expression (4); dashed-lines with markers: small-argument expression (9).

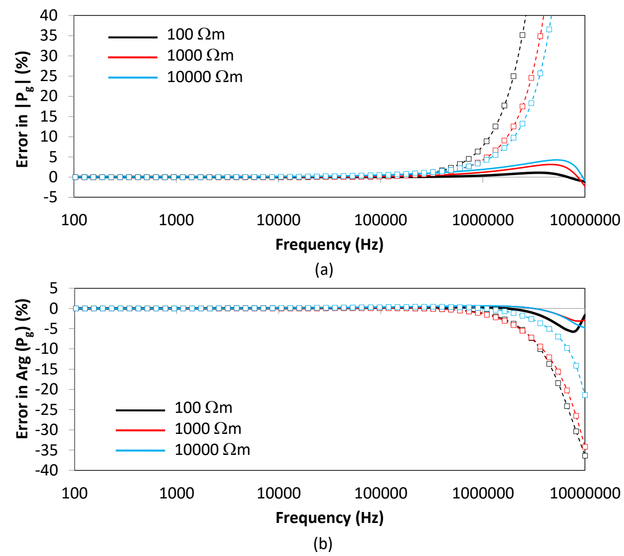

Fig. 5. Relative error in the calculation of element (3,1) of the ground-return potential coefficients matrix taking as reference Xue *et al.*'s integral equation [8]. Solid lines: closed-form expression (4); dashed-lines with markers: small-argument expression (9).

of $P_{g(1,1)}$, the maximum phase angle predicted by Xue *et al.*'s integral equation was considered. This was necessary to avoid a division by zero in the high-frequency range due to a zero crossing in the argument of $P_{g(m,n)}$ [16]. The calculations were again performed for $\rho = 100$, 1000 and 10000 Ωm.

The results in Fig. 4 demonstrate the high accuracy of (4) in the whole frequency range, regardless of the value of ground resistivity, with relative errors well below 5%. This confirms the results obtained in [16], but now considering frequency-dependent soil parameters. On the other hand, the small-argument expression (9) presents similar accuracy only up to 1 MHz or so. Greater errors are verified with (9) for the 100 Ωm soil. However, this problem is alleviated by the fact that for a low-resistivity soil the influence of the ground-return admittance on the calculation of the per-unit-length parameters of the cable is generally less important [6], [7], [12], [15].

The relative errors associated with the calculation of $P_{g(3,1)}$ with (4) and (9), taking as reference Xue *et al.*'s integral equations, are shown in Fig. 5. This element relates phases A and C in the ground-potential coefficients matrix. The calculations were performed as before, and a similar trend is observed, with (4) performing accurately in the whole frequency range, regardless of soil resistivity, and the small-argument approximation (9) losing accuracy above 1 MHz or so. Although not shown, the mutual term relating cables A and B (or, equivalently, B and C) behaves similarly.

IV. Time-Domain Analysis

The validity of the proposed small-argument approximations is further demonstrated in this section by considering switching transient studies in the 138-kV cable system shown in Fig. 1. The tested cases considered the mixed-mode excitation shown in Fig. 6(a) and the ground-mode excitation shown in Fig. 6(b). To excite a wide frequency range, a step voltage with 1 kV amplitude was applied. The calculation of the cable parameters including core losses, sheath losses, internal insulation and external jacket was performed in Matlab following the approach described in [22] with the data of Table I. As before, ground resistivity values of 100, 1000 and 10000 Ωm were considered according to the Alipio-Visacro model [19].

The transient calculations were performed with the universal line model (ULM) [23] implemented in the Alternative Transients Progam (ATP) as a foreign model [24]. Two cable lengths, 100 m and 1 km, were considered to illustrate the model response to different resonant frequencies associated with the multiple reflections occurring at the cable ends. The ground-return parameters were determined with Xue *et al.*'s integral equations [8], with De Conti *et al.*'s equations (1) and (4) [16], and with the small-argument expressions (8) and (9).

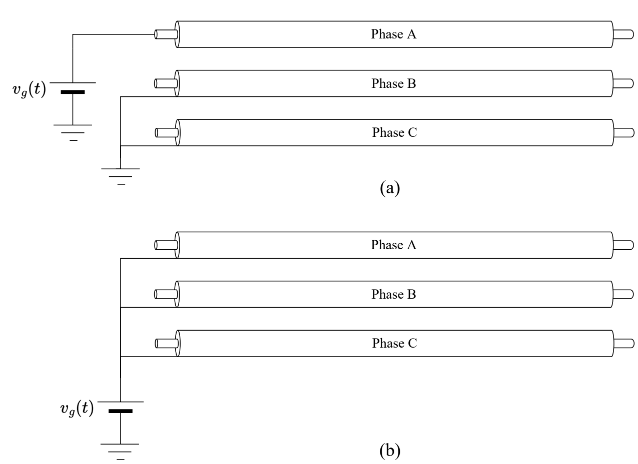

Fig. 6. Configurations for the transient simulations.

The fitting of the characteristic admittance and the propagation function required in the ULM was performed in Matlab using the vector fitting technique [25]. In the cases involving Xue *et al.*'s and De Conti *et al.*'s equations, the fitting was performed from 0.1 Hz to 10 MHz, except for $\rho = 10000$ Ωm, in which case the upper frequency was limited to 5 MHz to avoid instabilities in the time-domain simulations. For the small-argument expressions, the fitting was performed up to 1 MHz.

Figs. 7, 8, and 9 illustrate the core voltages calculated at the receiving ends of phases A and C for ρ =100, 1000 and 10000 Ωm, respectively, considering the configuration of Fig. 6(a) and different cable lengths. It is observed in the figures that the calculated voltages are coincident in all tested conditions. The good performance of the proposed small-argument expressions is verified even for the cable length of 100 m, which excites higher resonant frequencies due to the shorter oscillation period associated with successive reflections at the cable ends, regardless of the ground resistivity value. The performance of the small-argument expressions (8) and (9) is equivalent to (1) and (4) in this case, and consequently to Xue *et al.*'s equations [8].

Fig. 10 illustrates the core voltages at the receiving end of phase B for the ground-mode excitation of Fig. 6(b), for ρ = 100, 1000 and 10000 Ωm and cable lengths of 100 m and 1 km. In all cases, the voltages calculated with the different expressions are equivalent regardless of ground resistivity and cable length. Such a good agreement is related to the fact that the resonant frequencies estimated from the waveforms shown in Fig. 10 are well below the upper frequency limit of the small-argument approximations (8) and (9), even for the 100-m long cable. Since the ground-mode excitation is the one in which the ground-return impedance and admittance play the most significant role in terms of transient responses, the obtained results confirm not only the accuracy of the approximate expressions (1) and (4), but also of the small-argument approximations (8) and (9), even though the latter are strictly valid only up to 1 MHz or so.

V. Conclusions

This paper proposes small-argument approximations for the calculation of the ground-return impedance and admittance of underground cables. The proposed small-argument expressions are simplified versions of two closed-form expressions proposed by De Conti *et al.* [16] as an approximation to the integral equations of Xue *et al.* [8].

The validity of the proposed small-argument expressions is demonstrated through simulations both in the frequency domain and in the time domain for ground resistivity values ranging from 100 Ωm to 10000 Ωm and different cable lengths.

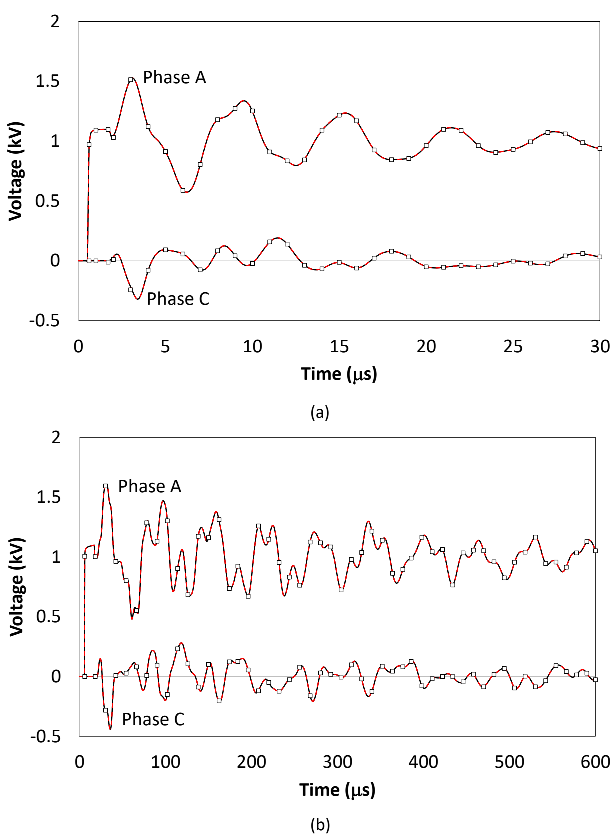

Fig. 7. Receiving-end voltages at the core of phases A and C for $\rho = 100$ Ωm and the mixed-mode excitation of 6(a). (a) 100-m long cables; (b) 1-km long cables. Black solid lines: Xue *et al.*'s equations [8]; red-dashed lines: De Conti *et al.*'s equations (1) and (4) [16]; square dots: small-argument approximations (8) and (9).

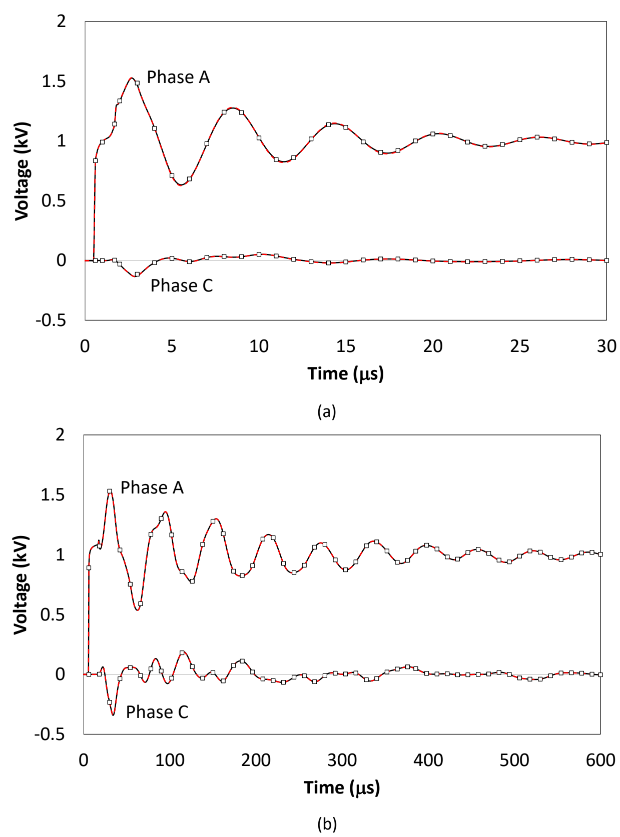

Fig. 8. Receiving-end voltages at the core of phases A and C for ρ = 1000 Ωm and the mixed-mode excitation of 6(a). (a) 100-m long cables; (b) 1-km long cables. Black solid lines: Xue *et al.*'s equations [8]; red-dashed lines: De Conti *et al.*'s equations (1) and (4) [16]; square dots: small-argument approximations (8) and (9).

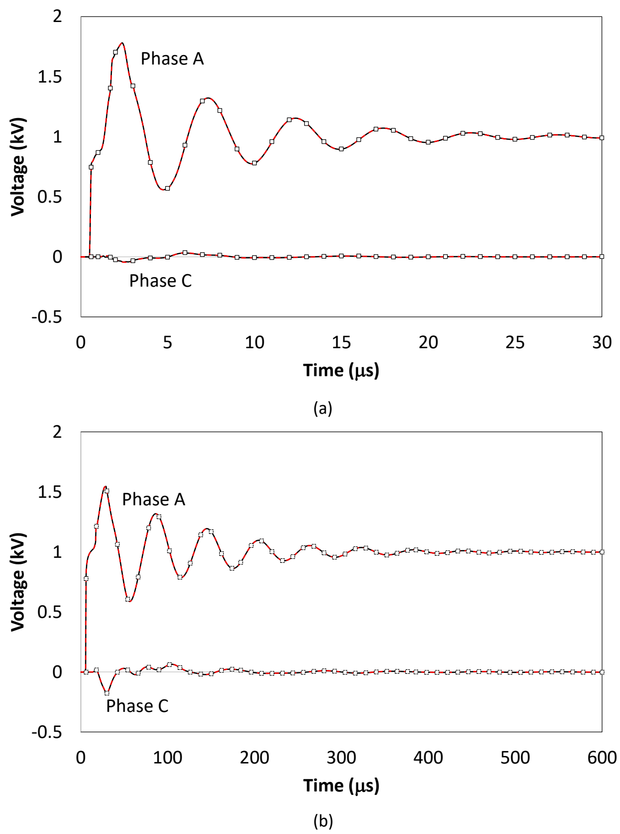

Fig. 9. Receiving-end voltages at the core of phases A and C for $\rho = 10000$ Ωm and the mixed-mode excitation of 6(a). (a) 100-m long cables; (b) 1-km long cables. Black solid lines: Xue *et al.*'s equations [8]; red-dashed lines: De Conti *et al.*'s equations (1) and (4) [16]; square dots: small-argument approximations (8) and (9).

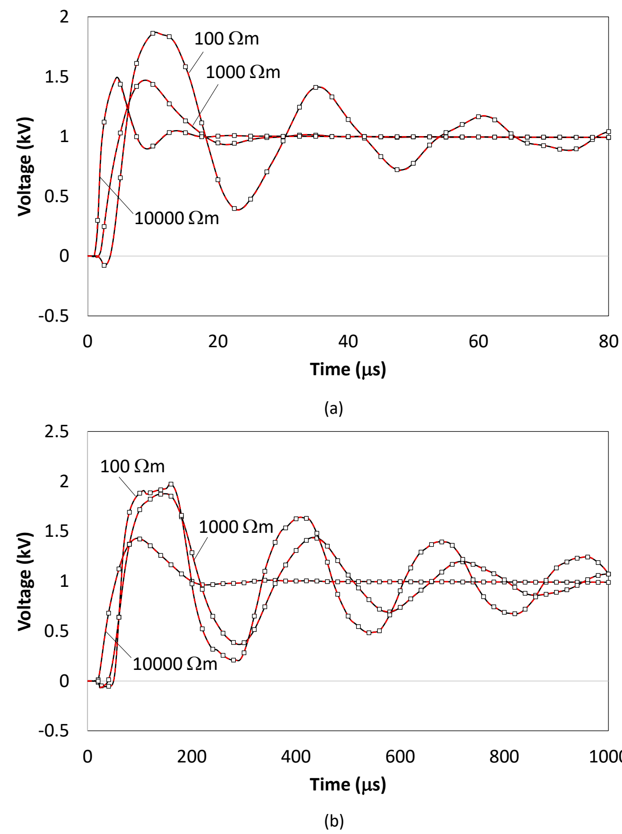

Fig. 10. Receiving-end voltages at the core of phase B for the ground-mode excitation of 6(b) and different values of ground resistivity. (a) 100-m long cables; (b) 1-km long cables. Black solid lines: Xue *et al.*'s equations [8]; red-dashed lines: De Conti *et al.*'s equations (1) and (4) [16]; square dots: small-argument approximations (8) and (9).

The ground-return parameters were calculated considering a realistic soil model that takes into consideration the frequency-dependent nature of both ground resistivity and permittivity. The proposed small-argument approximations are shown to perform accurately up to 1 MHz or so, which covers most phenomena of interest to underground cables. If a greater accuracy is required in the high-frequency range, the closed-form expressions proposed in [16] could be alternatively used.

The performed transient studies demonstrated that the proposed small-argument approximations lead to results that are equivalent to those obtained with the integral equations of Xue *et al.* [8] for different types of excitation and a wide range of ground resistivities. This suggests that the proposed expression can be conveniently used in switching transient studies without incurring in significant errors. The results also demonstrate the accuracy of the closed-form equations of De Conti *et al.* [16] for transient studies. Although a typical 138-kV single-core cable system with flat configuration was taken as reference, similar conclusions could be drawn for typical vertical and trefoil cable configurations.

The proposed expressions are simple and can be easily implemented in a computer code for the evaluation of the per-unit-length parameters of underground cable systems. More importantly, they can be easily typed in scientific calculators, which makes them particularly convenient for practitioners and non-academic users. For long cables and low-resistivity soils, an even simpler approach in which only the ground-return impedance obtained with the small-argument approximation (8) is considered and the ground-return admittance is completely neglected could lead to sufficiently accurate results in low-frequency transient studies [12], [17]. Although this approach has been considered in most electromagnetic transient simulation tools for many decades, it is not sufficiently general. Consequently, both the ground-return impedance and admittance should be considered for a consistent modeling of underground cables, especially for high values of ground resistivity and short cables [15].

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