T-Equivalent Zero-Sequence Impedances of Transformers with a Tertiary Delta Winding Obtained from Test Data

Mohammad Shafieipour, Robert Southey, Simon Fortin, Farid Paul Dawalibi

Abstract— **When computing short-circuit (i.e., fault) currents using full-wave electromagnetic or circuit-based methods that are not reduced to sequence components only, equivalent Thevenin source impedances are required at substation terminals or at the end of truncated transmission lines that explicitly define self and mutual parameters between all modelled conductors. This information is typically provided as sequence component equivalents, including zero-sequence components. For three-phase, three-winding transformers that have a tertiary delta winding, zero-sequence tests are typically performed with the delta winding in place. Results from such tests account for the circulating currents in the delta winding. To use the resulting impedances in a computer model based on a T-equivalent circuit, the test data must be transformed accordingly. In this article, formulae for such transformations are derived from the typical zero-sequence test setup vs. the desired zero-sequence test setup. The methodology is applicable to three-winding transformers with a tertiary delta winding such as star-star-delta and auto-star-delta configurations.**

Keywords: **Transformer T-equivalent zero-sequence circuit, T-circuit zero-sequence impedances, tertiary delta winding, three-winding transformer model, star-star-delta (Y-Y-Δ), auto-star-delta (Auto-Y-Δ).**

I. Introduction

THE T-equivalent circuit has been widely used for modeling three-winding power transformers [1]. Determining parameters of the positive-sequence T-equivalent circuit is straightforward, since three independent positive-sequence short-circuit tests can be used to obtain the three impedances forming the positive-sequence T-circuit. For zero-sequence T-equivalent circuit on the other hand, if a tertiary delta-winding is installed, results will be impacted by circulating currents. Hence, a total of four zero-sequence tests can be applied, two of which are not independent. In fact, in the IEEE standard [2], all four zero-sequence tests are defined, with one of them considered to be redundant and can be used as a check on the test results [3].

In this paper, we show that ignoring one out of four possible zero-sequence tests may be an over-simplification, since the two dependent tests may lead to an overdetermined system of equations with no unique solution. Subsequently, we provide practical solutions based on the available data and the expected physical behavior of a transformer in the presence of a tertiary delta winding in short-circuit simulations.

II. Problem Statement

Transformer factory acceptance tests (FAT) are typically performed once a transformer is built. For three-phase three-winding transformers that have a tertiary delta winding, such as star-star-delta (Y-Y-Δ) and auto-star-delta (Auto-Y-Δ) transformers, this means that the FATs are performed with the tertiary Δ winding and its connections in place. For positive-sequence tests, this does not impact the results. However, the zero-sequence short-circuit impedance tests are impacted by the circulating currents formed in the Δ winding. The zero-sequence T-equivalent circuit for such a transformer is depicted in Fig. 1 [3].

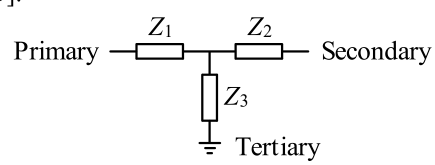

Fig. 1. Zero-sequence T-equivalent circuit with a tertiary Δ winding. As a result of the circulating currents formed in the Δ winding, Z_3 must be grounded.

Since it is common for the tertiary Δ winding to be inaccessible, the zero-sequence tests are typically performed by energizing the primary and secondary windings. This requires four different scenarios, as shown in Fig. 2.

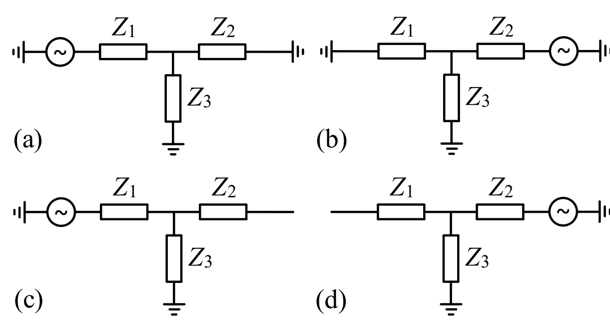

Fig. 2 T-equivalent circuits for typical zero-sequence short-circuit tests.

The results of these four zero-sequence tests are typically reported in the datasheet of a Y-Y-Δ or Auto-Y-Δ transformer $(Z0'_{12}, Z0'_{21}, Z0'_{13}, Z0'_{23})$[1] where:

- $Z0'_{12}$ is the measured primary-to-secondary zero-sequence

The authors are with SES & technologies ltd., Laval, QC H7L6E8, Canada (e-mail of corresponding author: Mohammad.Shafieipour@sestech.com).

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[1] Primed notation is used for the typically measured zero-sequence impedances ($Z0'_{12}, Z0'_{21}, Z0'_{13}, Z0'_{23}$), indicating that they cannot be directly used in the transformer T-equivalent model.

impedance (Fig. 2a),

- $Z0'_{21}$ is the measured secondary-to-primary zero-sequence impedance (Fig. 2b),
- $Z0'_{13}$ is the measured primary-to-tertiary zero-sequence impedance (Fig. 2c),
- $Z0'_{23}$ is the measured secondary-to-tertiary zero-sequence impedance (Fig. 2d).

However, from the modeling viewpoint, this is not directly applicable since the T-circuit should take a more general form to accommodate accessing all terminals: not only the primary and secondary windings, but also the tertiary winding, as shown in Fig. 3.

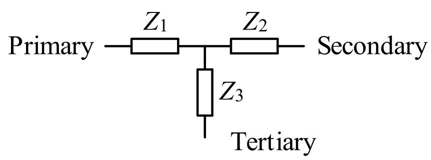

Fig. 3. Zero-sequence T-equivalent circuit, assuming that the tertiary winding is interrupted and, thus, no circulating current can form in it.

For such a circuit, which implies that the tertiary winding connections are removed or the delta loop is left open, if the primary winding is energized and the secondary winding is shorted (and vice versa), no circulating current forms in the tertiary winding during zero-sequence tests. This directly corresponds to the positive-sequence T-equivalent model which has the same circuit as Fig. 3 [4]. Therefore, unlike typical zero-sequence tests in which two different tests are required for these conditions (Fig. 2a and Fig. 2b), the zero-sequence T-circuit of Fig. 3 requires only one test for these conditions since ($Z0_{12} = Z0_{21}$)[2]. Therefore, three tests are sufficient to obtain the impedances ($Z0_{12}, Z0_{13}, Z0_{23}$), as shown in Fig. 4.

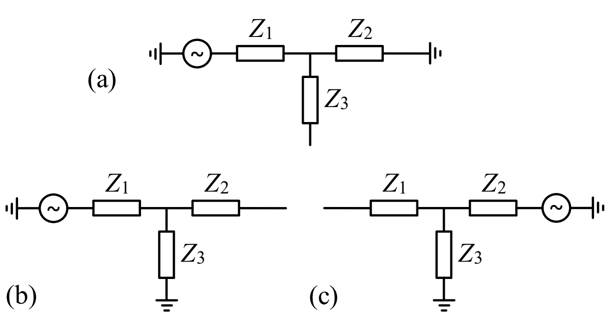

Fig. 4. T-equivalent circuits for the desired zero-sequence short-circuit tests.

It is now evident that the impedances obtained from the typical four zero-sequence tests in Fig. 2 ($Z0'_{12}, Z0'_{21}, Z0'_{13}, Z0'_{23}$) should be transformed into the three impedances that would be obtained from the desired three zero-sequence tests in Fig. 4 ($Z0_{12}, Z0_{13}, Z0_{23}$). This would correspond to zero-sequence tests performed prior to forming the delta connection when the three legs are interrupted, and no current can circulate between them. The need for this transformation is more pronounced in the following two scenarios:

- When the difference between $Z0'_{12}$ and $Z0'_{21}$ exceeds the anticipated measurement errors (e.g., >0.5%).
- When only three measurement values are available, specifically $Z0'_{13}$ and $Z0'_{23}$, but only one of $Z0'_{12}$ or $Z0'_{21}$ is available.

In such instances, there can be deviations between $Z0'_{12}$ and $Z0'_{21}$ that are not solely due to measurement or round-off errors. Rather, they are caused by circulating currents in the delta winding and should be considered in the computations. The resulting impedances can be used directly in a computer model based on the T-equivalent zero-sequence circuit simulation tool such as [5]. In this article, we derive formulations for such a transformation.

III. Formulation

A. Obtaining the T-Equivalent Zero-Sequence Impedances

The conversion involves calculating the T-equivalent zero-sequence impedances of Fig. 3 (Z_1, Z_2, Z_3). It is then easy to calculate the desired three inter-winding zero-sequence impedances as

$$\begin{aligned} \text{Primary-to-secondary impedance: } Z0_{12} &= Z_1 + Z_2, \\ \text{Primary-to-tertiary impedance: } Z0_{13} &= Z_1 + Z_3, \\ \text{Secondary-to-tertiary impedance: } Z0_{23} &= Z_2 + Z_3. \end{aligned} \tag{1}$$

According to the tests defined in Fig. 2, the three parameters Z_1, Z_2, and Z_3 should be calculated such that the following four equations are satisfied:

$$\begin{aligned} Z0'_{12} &= Z_1 + (Z_2 \parallel Z_3) && \text{(Equation I)}, \\ Z0'_{21} &= Z_2 + (Z_1 \parallel Z_3) && \text{(Equation II)}, \\ Z0'_{13} &= Z_1 + Z_3 && \text{(Equation III)}, \\ Z0'_{23} &= Z_2 + Z_3 && \text{(Equation IV)}. \end{aligned} \tag{2}$$

There are more equations than unknowns in the above system of equations, which leads to an overdetermined system. This requires simplification. This will be handled depending on the available test data as discussed in the following subsections.

1) Assuming $Z0'_{12}$, $Z0'_{21}$, $Z0'_{13}$, and $Z0'_{23}$ Are Available

Usually, primary-to-secondary $Z0'_{12}$ and secondary-to-primary $Z0'_{21}$ impedances are more important than the primary-to-tertiary $Z0'_{13}$ and secondary-to-tertiary $Z0'_{23}$ impedances in short-circuit simulations. Therefore, we take Equation I, Equation II, and Equation III with Equation IV of (2) to form a system of independent equations, as follows:

$$\begin{aligned} Z0'_{12} &= Z_1 + \frac{Z_2 \cdot Z_3}{Z_2 + Z_3}, \\ Z0'_{21} &= Z_2 + \frac{Z_1 \cdot Z_3}{Z_1 + Z_3}, \\ Z0'_{13} + Z0'_{23} &= Z_1 + Z_2 + 2Z_3. \end{aligned} \tag{3}$$

By solving the above system, we expect that $Z0'_{12}$ and $Z0'_{21}$ are enforced accurately, while $Z0'_{13}$ and $Z0'_{23}$ will be as accurate as any test data discrepancies or round-off errors allow (this is a result of having an overdetermined system of

[2] Unprimed notation is used for the desired zero-sequence impedances ($Z0_{12}, Z0_{13}, Z0_{23}$), indicating that they can be directly used in the transformer T-equivalent model.

equations). This leads to quadratic equations, and thus, it has two sets of solutions. The solutions are as follows:

- Solution #1

$$Z_1 = \frac{ac+ad+\sqrt{ab(-ac-ad+c^2-cb+2cd-bd+d^2)}}{a+b}, \tag{4}$$

$$Z_2 = \frac{cb+bd+\sqrt{ab(-ac-ad+c^2-cb+2cd-bd+d^2)}}{a+b}, \tag{5}$$

$$Z_3 = \frac{-\sqrt{-ab(c+d)(a-c+b-d)}}{a+b}. \tag{6}$$

- Solution #2

$$Z_1 = \frac{ac+ad-\sqrt{ab(-ac-ad+c^2-cb+2cd-bd+d^2)}}{a+b}, \tag{7}$$

$$Z_2 = \frac{cb+bd-\sqrt{ab(-ac-ad+c^2-cb+2cd-bd+d^2)}}{a+b}, \tag{8}$$

$$Z_3 = \frac{\sqrt{-ab(c+d)(a-c+b-d)}}{a+b}. \tag{9}$$

where:

$$a = Z0'_{12},\ b = Z0'_{21},\ c = Z0'_{13},\ d = Z0'_{23} \tag{10}$$

Both solutions are mathematically valid, but we should only select one of them. Section III-B explains how this selection can be made.

2) Assuming $Z0'_{12}$, $Z0'_{13}$, and $Z0'_{23}$ Are Available, but $Z0'_{21}$ Is Not Available

In this case, we can simply drop Equation II from (2) and solve the resulting (3×3) system of equations. The solutions are as follows:

- Solution #1

$$Z_1 = c - \sqrt{d(c-a)}, \tag{11}$$

$$Z_2 = d - \sqrt{d(c-a)}, \tag{12}$$

$$Z_3 = \sqrt{d(c-a)}. \tag{13}$$

- Solution #2

$$Z_1 = c + \sqrt{d(c-a)}, \tag{14}$$

$$Z_2 = d + \sqrt{d(c-a)}, \tag{15}$$

$$Z_3 = -\sqrt{d(c-a)}. \tag{16}$$

where *a*, *c*, and *d* are defined in (10), and the correct solution is selected based on the physical grounds explained in Section III-B.

It is worth noting that, due to the symmetry in both circuits of Fig. 1 and Fig. 3, the same expressions are applicable if $Z0'_{21}$ is available instead of $Z0'_{12}$. Simply swap the primary and secondary winding locations in the T-circuit, and the rest of the procedure applies as is.

B. Selecting a Solution

1) Selecting a Solution Based on Positive-Sequence Impedances

First, we compute two sets of inter-winding zero-sequence impedances ($|Z0_{12}|, |Z0_{13}|, |Z0_{23}|$) for each solution using (1) where Z_1, Z_2, Z_3 are obtained from (4)-(9) or (11)-(16) depending on the available test data. Then, we compare the magnitude of these results with that of the magnitude of the positive-sequence impedances ($|Z1_{12}|, |Z1_{13}|, |Z1_{23}|$). The set of solutions that has the smaller differences is selected and used in the computer model that is based on the zero-sequence T-circuit. The difference is computed as $(||Z0_{12}| - |Z1_{12}|| + ||Z0_{13}| - |Z1_{13}|| + ||Z0_{23}| - |Z1_{23}||)$ for each set of inter-winding zero-sequence impedances. This is because the positive- and zero-sequence impedances of the three branches of a T-equivalent circuit of a typical transformer are expected to be of the same order of magnitude.

2) Selecting a Solution Based on the Computed Z_3

Let us begin with Equation III of (2). Solving for Z_1, we get:

$$Z_1 = Z0'_{13} - Z_3. \tag{17}$$

Similarly, using Equation IV of (2), we can write:

$$Z_2 = Z0'_{23} - Z_3. \tag{18}$$

By adding (17) and (18), we get:

$$Z_1 + Z_2 = Z0'_{13} + Z0'_{23} - 2Z_3. \tag{19}$$

From (6) and (9), we conclude that Z_3 is either a positive or negative reactance, ignoring the small resistive component. Now, from physical principles (we are measuring the series impedance of a three-winding transformer for the corresponding tests), $Z0'_{13}$ and $Z0'_{23}$ will always have large positive reactances compared with other measured transformer impedances. Therefore, it is reasonable to expect $Z_1 + Z_2$ (i.e., the high-low impedance of the transformer with the delta tertiary interrupted) to be smaller than the sum of the relatively large high-to-tertiary $Z0'_{13}$ and low-to-tertiary $Z0'_{23}$ impedances.

$$Z_1 + Z_2 < Z0'_{13} + Z0'_{23}. \tag{20}$$

From (19) and (20), it is shown that we must choose the solution set that results in a positive value for the Z_3 reactance. Once the correct set of solutions for Z_1, Z_2, Z_3 is selected, (1) is used to calculate the desired zero-sequence impedances that can be directly used in the T-circuit-based computer model.

IV. Simplified Solutions

By comparing the circuits in Fig. 2c and Fig. 2d with those of Fig. 4b and Fig. 4c, respectively, it is clear that the typical zero-sequence tests and desired zero-sequence tests are identical when the tertiary winding is short-circuited. Thus, we have

$$Z0_{13} = Z0'_{13}, \quad Z0_{23} = Z0'_{23}. \tag{21}$$

Hence, the measured zero-sequence primary-to-tertiary and secondary-to-tertiary impedances $(Z0'_{13}, Z0'_{23})$ may be directly used in the model. Again, results will be as accurate as any test data discrepancies or round-off errors. When all $Z0'_{12}$, $Z0'_{21}$, $Z0'_{13}$, and $Z0'_{23}$ are available, the primary-to-secondary impedance is calculated as

$$Z0_{12} = Z_1 + Z_2 = (Z0'_{13} + Z0'_{23}) \pm \frac{2\left\{\sqrt{Z0'_{12}Z0'_{21}[(Z0'_{13}+Z0'_{23})^2-(Z0'_{12}+Z0'_{21})(Z0'_{13}+Z0'_{23})]}\right\}}{Z0'_{12}+Z0'_{21}}. \tag{22}$$

When $Z0'_{21}$ is not available, the primary-to-secondary impedance is obtained as

$$Z0_{12} = Z_1 + Z_2 = (Z0'_{13} + Z0'_{23}) \pm 2\left\{\sqrt{Z0'_{23}(Z0'_{13} - Z0'_{12})}\right\}. \quad (23)$$

Depending on the available data, results for both signs can be computed in (22) or (23). Subsequently, the computed complex value whose magnitude is closer to the magnitude of the positive-sequence primary-to-secondary impedance $Z1_{12}$ (see Section III-B1) should be chosen. Alternatively, the solution based on the calculated winding impedance Z_3 (see Section III-B2) should be selected.

V. Numerical Results

As mentioned earlier, the methodology is applicable to Y-Y-Δ and Auto-Y-Δ configurations. For conciseness, we only present numerical results for the Y-Y-Δ case. It is expected that similar results are produced for a transformer with Auto-Y-Δ configuration.

TABLE I provides details of the transformer under study. Such data is typically provided by the manufacturer (e.g., FAT report, nameplate, etc.). Using all parameters given in TABLE I is straightforward in a T-circuit-based computer model, except for the measured zero-sequence impedances highlighted in the table. In this section, we apply the presented methodology to this example by first computing the pertinent inter-winding impedances and using them in a computer model to produce expected short-circuit simulation results.

TABLE I
Details of the Y-Y-Δ transformer under consideration.

Three-phase power rating	*P*	150	MVA
Primary winding (Y) rated voltage	*V1*	220	kV
Secondary winding (Y) rated voltage	*V2*	115	kV
Tertiary winding (Δ) rated voltage	*V3*	14	kV
Positive-sequence primary-to-secondary impedance	$Z1_{12}$	0.21 + 11.12j	%
Positive-sequence primary-to-tertiary impedance	$Z1_{13}$	0.315 + 8.77j	%
Positive-sequence secondary-to-tertiary impedance	$Z1_{23}$	0.32 + 4.3j	%
Measured zero-sequence primary-to-secondary impedance	$Z0'_{12}$	0.34 + 9.5j	%
Measured zero-sequence secondary-to-primary impedance	$Z0'_{21}$	0.18 + 5.7j	%
Measured zero-sequence primary-to-tertiary impedance	$Z0'_{13}$	1.1 + 26.3j	%
Measured zero-sequence secondary-to-tertiary impedance	$Z0'_{23}$	0.77 + 15.2j	%
Excitation current	*Iexc*	0.14	%
No-load loss	*NLL*	32000	W
Vector group	YNyn0d1		
Core type	Three-limb		

A. Calculating the Desired Zero-Sequence Impedances $Z0_{12}, Z0_{13}, Z0_{23}$

We apply the presented methods and calculate the desired three zero-sequence parameters ($Z0_{12}, Z0_{13}, Z0_{23}$) that can be used in the zero-sequence T-equivalent circuit model. Results are tabulated in TABLE II. All methods have produced similar results, with small deviations. It is worth noting that both selection methods introduced in Section III-B lead to the same decisions in all cases.

TABLE II
Calculated zero-sequence impedances using the presented methods.

Applied Method	Section III-A1	Section III-A2	Eq. (22)	Eq. (23)	
$Z0_{12}$	0.345 + 9.511j	0.337 + 9.539j	0.345 + 9.511j	0.337 + 9.539j	%
$Z0_{13}$	1.209 + 25.93j	1.1 + 26.3j	1.1 + 26.3j	1.1 + 26.3j	%
$Z0_{23}$	0.660 + 15.56j	0.77 + 15.2j	0.77 + 15.2j	0.77 + 15.2j	%

B. Simulating the Zero-Sequence Short-Circuit Tests

To study the impact of the differences between the calculated zero-sequence impedances in TABLE II, we model the four short-circuit zero-sequence tests shown in Fig. 2 in the HIFREQ computation module of the CDEGS-MultiFields software package [5]. However, it is expected that the same procedure is applicable to any T-circuit-based computer model as long as both the positive-sequence and zero-sequence scenarios are included in the computer model. Note that the T-equivalent impedances (in %) are converted to series resistance (in Ω) and inductance (in Henry) branches in the computer model using $Z=R+j\omega L$ where $\omega=2\pi f$ is the angular frequency.

The simulated impedances are calculated as $Z = V/I$, where V is the single-phase rated winding voltage and I is the longitudinal current in the conductor connected to the energized winding. Results are given in TABLE III where percentage of the magnitude of the impedances are reported to facilitate easier comparison.

TABLE III
Results of the zero-sequence tests simulated in HIFREQ using the calculated zero-sequence impedances given in Table II.

	TABLE I	Section III-A1	Section III-A2	Eq. (22)	Eq. (23)	
$\lvert Z0'_{12}\rvert$	9.506	9.508	9.508	9.478	9.508	%
$\lvert Z0'_{21}\rvert$	5.702	5.708	5.509	5.492	5.509	%
$\lvert Z0'_{13}\rvert$	26.32	25.98	26.36	26.36	26.36	%
$\lvert Z0'_{23}\rvert$	15.21	15.62	15.26	15.26	15.26	%

All models have produced satisfactory results when compared with the results of TABLE I as the reference solution. In fact, for most practical purposes, all four models can be used interchangeably. However, it is worth noting that the method of Section III-A1 (highlighted column) has produced more accurate results for $|Z0'_{12}|$ and $|Z0'_{21}|$ but less accurate results for $|Z0'_{13}|$ and $|Z0'_{23}|$. This is expected, since this model enforces both primary-to-secondary and secondary-to-primary parameters, while taking the average of the primary-to-tertiary and secondary-to-tertiary parameters (3). Therefore, the most suitable method can be selected based on the available data and which inter-winding impedance is more important for a particular study.

VI. Conclusion

The paper covers the mathematical formulation and application of different techniques to obtain T-equivalent zero-sequence impedances from typical zero-sequence test data. Results can be directly used in a computer model based on the

T-equivalent circuit model featuring both zero-sequence and positive-sequence T-equivalent circuits. Numerical results demonstrate the validity of the method both in calculating the equivalent inter-winding zero-sequence impedances as well as reproducing the results of the typical zero-sequence short-circuit tests.

VII. References

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